

On-shell methods for NLO calculations

Ralf Sattler

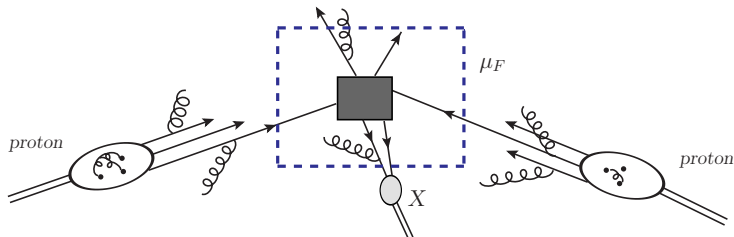
HU Berlin & DESY Zeuthen



Autumn Block Course 2009

the modell of Hadron scattering

Factorisation of the (high energy) scattering process into Partonevolution und -scattering [COLLINS,SOPER,STERMAN]



subject of the following slides: **Parton cross section** σ_{Parton}

perturbative expansion and corrections

$$\sigma_{Parton} = [\hat{\sigma}_0(\alpha_S^z) + \hat{\sigma}_1(\alpha_S^{z+1}) + \dots] \quad \alpha_S = \alpha_S(\mu_R^2)$$

perturbative expansion and corrections

$$\sigma_{Parton} = [\hat{\sigma}_0(\alpha_S^z) + \hat{\sigma}_1(\alpha_S^{z+1}) + \dots] \quad \alpha_S = \alpha_S(\mu_R^2)$$

1. virtual corrections

⇒ reduction and determination of the scale dependency

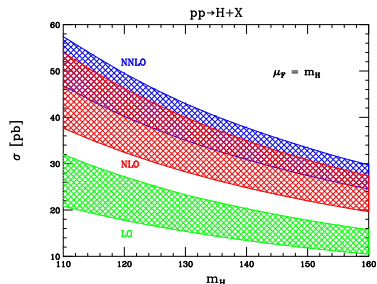
2. final state Bremsstrahlung

⇒ infrared safety of observables

perturbative expansion and corrections

$$\sigma_{Parton} = [\hat{\sigma}_0(\alpha_S^z) + \hat{\sigma}_1(\alpha_S^{z+1}) + \dots] \quad \alpha_S = \alpha_S(\mu_R^2)$$

example: Higgs production ($\mu_F = \mu_R = m_H$)



[ANASTASIOU, MELNIKOV, PETRIELLO]

performing a calculation

1. the naive way: draw all Feynman diagrams
(too small pieces, too much sums at one time)

performing a calculation

1. the naive way: draw all Feynman diagrams
(too small pieces, too much sums at one time)
2. the advanced approach: Feynman 2.0

performing a calculation

1. the naive way: draw all Feynman diagrams
(too small pieces, too much sums at one time)
2. the advanced approach: Feynman 2.0
 - **colour ordering** (separation of Lorentz and colour structure)

[MANGANO, PARKE]

performing a calculation

1. the naive way: draw all Feynman diagrams
(too small pieces, too much sums at one time)
2. the advanced approach: Feynman 2.0
 - **colour ordering** (separation of Lorentz and colour structure)
[MANGANO, PARKE]
 - **spinor helicity formalism** [XU ET. AL.]

performing a calculation

1. the naive way: draw all Feynman diagrams
(too small pieces, too much sums at one time)
2. the advanced approach: Feynman 2.0
 - **colour ordering** (separation of Lorentz and colour structure)
[MANGANO, PARKE]
 - **spinor helicity formalism** [XU ET. AL.]
3. the improved version: on-shell methods [BCFW, BDK]

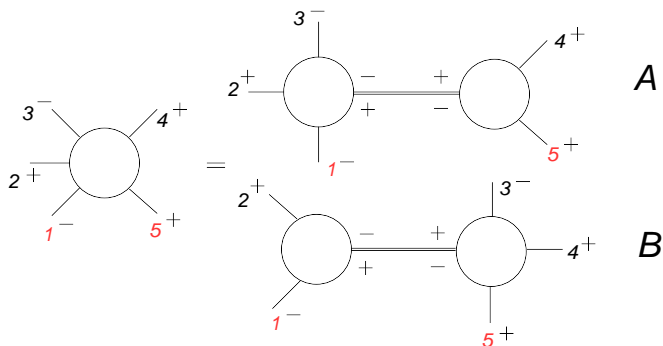
performing a calculation

1. the naive way: draw all Feynman diagrams
(too small pieces, too much sums at one time)
2. the advanced approach: Feynman 2.0
 - **colour ordering** (separation of Lorentz and colour structure)
[MANGANO, PARKE]
 - **spinor helicity formalism** [XU ET. AL.]
3. the improved version: on-shell methods [BCFW, BDK]
 - use **Cauchy's theorem** to solve integrals

performing a calculation

1. the naive way: draw all Feynman diagrams
(too small pieces, too much sums at one time)
2. the advanced approach: Feynman 2.0
 - **colour ordering** (separation of Lorentz and colour structure)
[MANGANO, PARKE]
 - **spinor helicity formalism** [XU ET. AL.]
3. the improved version: on-shell methods [BCFW, BDK]
 - use **Cauchy's theorem** to solve integrals
 - use **treelevel amplitudes** instead of vertices **as building blocks**

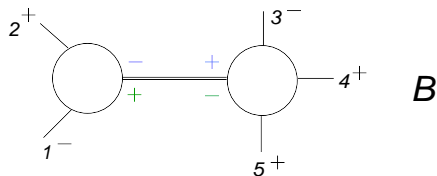
a treelevel example: 5 pt. gluon function



$$\hat{p}_1(z) = \lambda_1 (\tilde{\lambda}_1 - z \tilde{\lambda}_5)$$

$$\hat{p}_5(z) = (\lambda_1 + z \lambda_5) \tilde{\lambda}_5$$

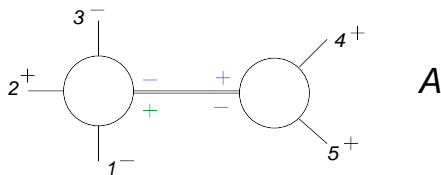
a treelevel example: 5 pt. gluon function



pole of contribution B: $z_{1,2} = \frac{[1\ 2]}{[5\ 2]}$

denominator: $\langle 3|1|2\rangle - z_{1,2} \langle 3\ 1\rangle [5\ 2] = 0$

a treelevel example: 5 pt. gluon function



pole of contribution A: $z_{4,5} = -\frac{\langle 54 \rangle}{\langle 14 \rangle}$

$$\begin{aligned}
 A_5^{2-} &= -i \frac{\langle 13 \rangle^4 [45]^3}{\langle 12 \rangle \langle 23 \rangle \langle 1|5|4 \rangle \langle 3|4|5 \rangle \langle 4|5|4 \rangle} \\
 &= i \frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}
 \end{aligned}$$

massive fermions and on shell methodes

they suffer from:

- a more difficult *inner* colour ordering of fermions
- less symmetries \rightarrow no vanishing helicity amplitudes
- less compact representation in terms of Weyl spinors

massive fermions and on shell methodes

they suffer from:

- a more difficult *inner* colour ordering of fermions
- less symmetries \rightarrow no vanishing helicity amplitudes
- less compact representation in terms of Weyl spinors

remaining properties:

- parity condition for involved gluons (factor 2)
- spin axis quantization freedom of heavy quarks (factor 4)

the situation at one loop

1.

$$A^{1-loop} = D(\epsilon) + \mathcal{F}$$

the situation at one loop

1.

$$A^{1-loop} = D(\epsilon) + \mathcal{F}$$

2. the scalar integralbase at one loop [f.e. ELLIS, ZANDERIGHI]

$$A^{1-loop} \sim d_{ijkl} \text{ (square)} + c_{ijk} \text{ (triangle)} + b_{ij} \text{ (bubble)} + a_i \text{ (tadpole)}$$

(a result of 30 years hard work in perturbative QFT)

⇒ remaining task: compute the coefficients

the situation at one loop

1.

$$A^{1-loop} = D(\epsilon) + \mathcal{F}$$

2. the scalar integralbase at one loop [f.e. ELLIS, ZANDERIGHI]

$$A^{1-loop} \sim d_{ijkl} \text{ (square)} + c_{ijk} \text{ (triangle)} + b_{ij} \text{ (bubble)} + a_i \text{ (tadpole)}$$

(a result of 30 years hard work in perturbative QFT)

⇒ remaining task: compute the coefficients

3. additional *inner* colour ordering due to loop contributions
→ primitive amplitudes

unitarity cuts I - foundations

consequences of S-matrix theory:

1. optical theorem

$$SS^\dagger = \mathbb{1} \xrightarrow{1 \text{ loop}} 2i \operatorname{Im} \left[\text{Diagram} \right] = \text{Diagram}$$

2. discontinuities of analytical continued matrix elements

$$2i \operatorname{Im}[\mathcal{M}(s)] = \operatorname{Disc}[\mathcal{M}(s)]$$

unitarity cuts I - foundations

consequences of S-matrix theory:

1. optical theorem

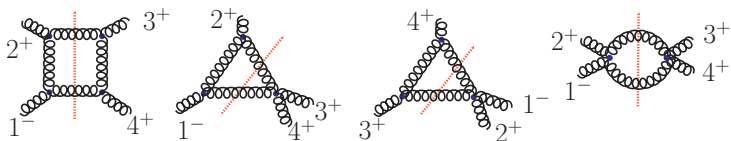
$$SS^\dagger = \mathbb{1} \xrightarrow{1 \text{ loop}} 2i \operatorname{Im} \left[\text{Diagram} \right] = \text{Diagram}$$

2. discontinuities of analytical continued matrix elements

$$2i \operatorname{Im}[\mathcal{M}(s)] = \operatorname{Disc}[\mathcal{M}(s)]$$

Sewing of treelevel amplitudes to 1-loop amplitudes allows for the calculation of the logarithmic parts of \mathcal{F} .

example: a $gg \rightarrow gg$ 1 loop configuration



unitarity cuts II - recent methodes

- the basic step of the calculation:

$$\frac{1}{l^2 + i\epsilon} \rightarrow \pi\delta^{(+)}(l^2) \quad (\text{Cutkosky rules})$$

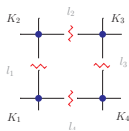
unitarity cuts II - recent methodes

- the basic step of the calculation:

$$\frac{1}{l^2 + i\epsilon} \rightarrow \pi \delta^{(+)}(l^2) \quad (\text{Cutkosky rules})$$

- implementation of multiple cuts (generalized unitarity)

[BRITTO,FENG]; [OPP]; [FORDE]



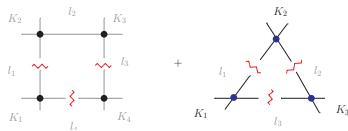
unitarity cuts II - recent methodes

- the basic step of the calculation:

$$\frac{1}{l^2 + i\epsilon} \rightarrow \pi \delta^{(+)}(l^2) \quad (\text{Cutkosky rules})$$

- implementation of multiple cuts (generalized unitarity)

[BRITTO,FENG]; [OPP]; [FORDE]



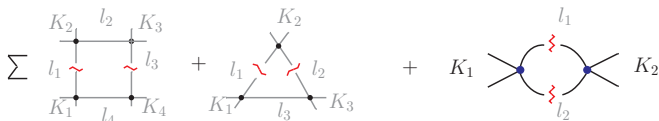
unitarity cuts II - recent methodes

- the basic step of the calculation:

$$\frac{1}{l^2 + i\epsilon} \rightarrow \pi \delta^{(+)}(l^2) \quad (\text{Cutkosky rules})$$

- implementation of multiple cuts (generalized unitarity)

[BRITTO,FENG]; [OPP]; [FORDE]



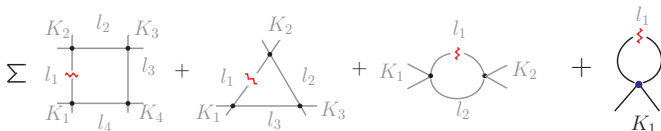
unitarity cuts II - recent methodes

- the basic step of the calculation:

$$\frac{1}{l^2 + i\epsilon} \rightarrow \pi \delta^{(+)}(l^2) \quad (\text{Cutkosky rules})$$

- implementation of multiple cuts (generalized unitarity)

[BRITTO,FENG]; [OPP]; [FORDE]



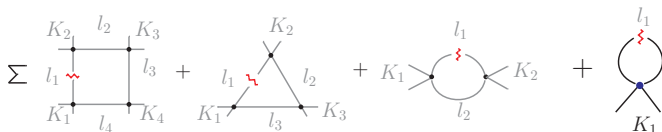
unitarity cuts II - recent methodes

- the basic step of the calculation:

$$\frac{1}{l^2 + i\epsilon} \rightarrow \pi \delta^{(+)}(l^2) \quad (\text{Cutkosky rules})$$

- implementation of multiple cuts (generalized unitarity)

[BRITTO,FENG]; [OPP]; [FORDE]



- a step further: generalizing to d dimensional loop momenta

[BERN, MORGAN]; [ANASTASIOU, BRITTO, FENG, KUNSZT, MASTROLIA]; [BADGER]

parametrisation by Forde

- loop momentum is an object with *four* degrees of freedom
→ choose a fixed base with complex momenta

$$2l^\mu = \alpha_1 \langle k_1 | \gamma^\mu | k_1 \rangle + \alpha_2 \langle k_2 | \gamma^\mu | k_2 \rangle + \\ \alpha_3 \langle k_1 | \gamma^\mu | k_2 \rangle + \alpha_4 \langle k_2 | \gamma^\mu | k_1 \rangle$$

parametrisation by Forde

- loop momentum is an object with *four* degrees of freedom
→ choose a fixed base with complex momenta

$$2l^\mu = \alpha_1 \langle k_1 | \gamma^\mu | k_1 \rangle + \alpha_2 \langle k_2 | \gamma^\mu | k_2 \rangle + \\ \alpha_3 \langle k_1 | \gamma^\mu | k_2 \rangle + \alpha_4 \langle k_2 | \gamma^\mu | k_1 \rangle$$

- unfixed coefficients → complex parameters

parametrisation by Forde

- loop momentum is an object with *four* degrees of freedom
→ choose a fixed base with complex momenta

$$2l^\mu = \alpha_1 \langle k_1 | \gamma^\mu | k_1 \rangle + \alpha_2 \langle k_2 | \gamma^\mu | k_2 \rangle + \\ \alpha_3 \langle k_1 | \gamma^\mu | k_2 \rangle + \alpha_4 \langle k_2 | \gamma^\mu | k_1 \rangle$$

- **unfixed coefficients** → **complex parameters**
- analytically cumbersome for double and single cut

parametrisation by Forde

- loop momentum is an object with *four* degrees of freedom
→ choose a fixed base with complex momenta

$$2l^\mu = \alpha_1 \langle k_1 | \gamma^\mu | k_1 \rangle + \alpha_2 \langle k_2 | \gamma^\mu | k_2 \rangle + \\ \alpha_3 \langle k_1 | \gamma^\mu | k_2 \rangle + \alpha_4 \langle k_2 | \gamma^\mu | k_1 \rangle$$

- **unfixed coefficients** → **complex parameters**
- analytically cumbersome for double and single cut
- numerically instabilities in a naive implementation

conclusions

1. physics @ LHC \Rightarrow cross sections @ NLO
2. on-shell methods
 - are promising tools for this task
 - have high potential for further progress

backup slides

factorization

task:

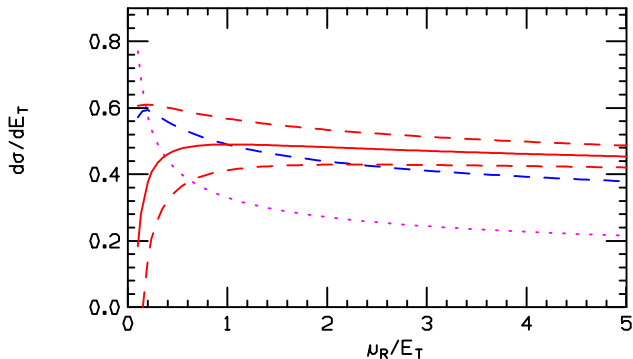
- separation of intrinsic processes from scattering events
- description for the time evolution of the intrinsic processes

$$\sigma_{pp \rightarrow X} = \sum_{r,s,t} \int dx_r dx_s f(x_r, \mu_F^2) f(x_s, \mu_F^2) \times \hat{\sigma}_{rs \rightarrow t} \times D(x_t, \mu_F^2)_{t \rightarrow X}$$

where

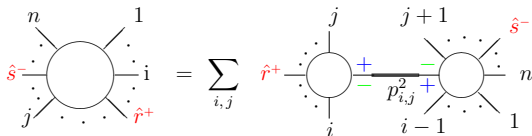
$$\hat{\sigma}_{rs \rightarrow t} = \left[\hat{\sigma}_0 + \alpha_s(\mu_R^2) \hat{\sigma}_1 + \dots \right]_{rs \rightarrow t}$$

scale dependence



Scale dependence for single jet production (inkl.) at $E_T = 100$ GeV. Theoretical predictions at LO (dotted curve), NLO (blue dashed) and NNLO (different models in red). [GLOVER 02]

general form of the recursion



$$A(z=0) = \sum_{i,j,h} A(i, \dots, \hat{r}^+, \dots, j, -\hat{P}_{i,j}^h) \frac{-i}{p_{i,j}^2} A(\hat{P}_{i,j}^{-h}, j+1, \dots, \hat{s}^-, \dots, i-1)$$

$$P_{i,j} = p_i + p_{i+1} + \dots + \hat{p}_r(z_{i,j}) + \dots + p_{j-1} + p_j$$