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Lattice study of pion pion scattering

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on behalf of **ETMC**



Scattering: from ...
Pion pion scattering
Motivation of Lattice QCD
Lüscher's method
 $l=2$ channel
 $l=1$ channel
Conclusion

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1 Scattering: from experiment to theory

- scattering is a main method in studying hadron-hadron interactions
- cross section σ are measured in experiment
- in the limit of $r \rightarrow \infty$, wave function $\psi(\vec{r})$ for two-particle state is expanded

$$\psi(\vec{r}) \rightarrow e^{i\vec{k}\cdot\vec{r}} + A(k, \theta)r^{-1}e^{ikr}$$

- σ can be given by scattering amplitude $A(k, \theta)$

$$\frac{d\sigma}{d\Omega} = A^2(k, \theta)$$

- according to partial wave analysis

$$A(k, \theta) = \frac{1}{k} \sum_{l=0}^{\infty} (2l+1) A_l(k) P_l(\cos \theta), \quad A_l(k) = \frac{e^{2i\delta_l(k)} - 1}{2i}$$

- in the limit of $k \rightarrow 0$, scattering length a_l is extracted from $\delta_l(k)$

$$\delta_l(k) = a_l k^{2l+1} + O(k^{2l+3})$$

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2 Pion pion scattering

- chiral symmetry spontaneously breaking: three Goldstone meson π^\pm, π^0
- property of π as Goldstone boson:
chiral symmetry play a central role in $\pi\pi$ scattering
- in χ -PT theory, chiral expansion of scattering amplitude $A(s)$ is given by:

$$A(s) = A(s)_2 + A(s)_4 + A(s)_6 + \dots$$

- we need low energy constants (LECs) to determine $A(s)$
- $A(s)$ is specified by isospin $A^I(s)$
- single pion: isospin triplet

$$\begin{aligned} |\pi^+\rangle &= |I = 1, I_z = +1\rangle, \\ |\pi^0\rangle &= |I = 1, I_z = 0\rangle, \\ |\pi^-\rangle &= |I = 1, I_z = -1\rangle. \end{aligned}$$

- two pion system:

$$\begin{aligned} |\pi\rangle \otimes |\pi\rangle &\equiv |\pi\pi\rangle \\ |I = 1\rangle \otimes |I = 1\rangle &= |I = 2\rangle \oplus |I = 1\rangle \oplus |I = 0\rangle \end{aligned}$$



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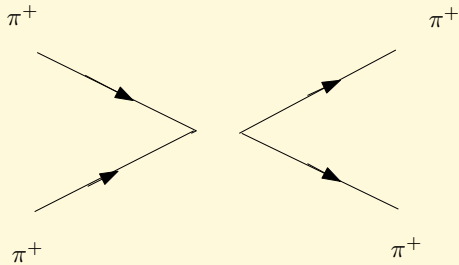
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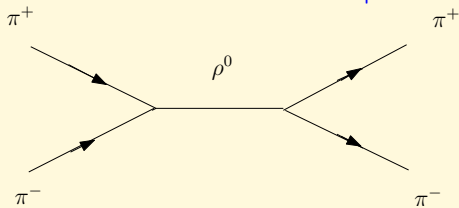
- I=2 channel, here we consider

$$|I = 2, I_z = +2\rangle = |\pi^+\pi^+\rangle$$



- ★ no resonance appears in this channel
- ★ LECs is required to determine χ -PT scattering amplitude

- I=1 channel: $|I = 1, I_z = 0\rangle = |\pi^+\pi^- - \pi^-\pi^+\rangle$



- ★ $\rho(770)$ resonance appears in this channel

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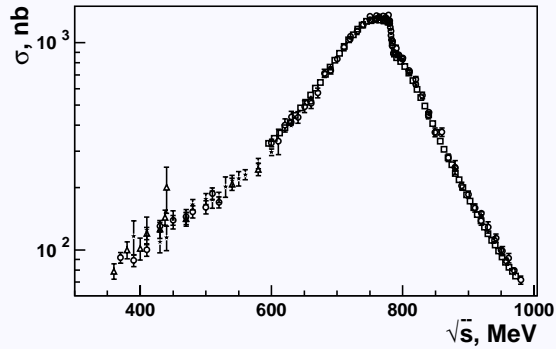
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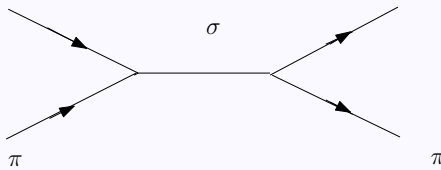


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resonance mass: $m_\rho=775$ MeV, large width: $\Gamma_\rho=149$ MeV:

- I=0 channel: $|I=0, I_z=0\rangle = |\pi^+\pi^- + \pi^-\pi^+ - \pi^0\pi^0\rangle$



- ★ $\sigma(600)$ resonance appears in this channel
- ★ $m_\sigma=513(32)$ MeV, $\Gamma_\sigma=670(134)$ MeV [CLEO Collaboration, 2002] ??
- ★ width is as large as mass \rightarrow
strong overlap between resonance and background
- ★ experimental existence for $\sigma(600)$ is not fully settled

$$m_\sigma = 400 - 1200 \text{ MeV}, \quad \Gamma_\sigma = 500 - 1000 \text{ MeV}. \quad [\text{PDG}]$$

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3 Motivation of Lattice QCD

- pros
 - ★ $\pi\pi$ scattering is **non-perturbative** in nature at low energies
 - ★ **LQCD** offers a non-perturbative method to study $\pi\pi$ scattering from first principle theory, QCD
 - * extract parameters of $\delta_l^I(\sqrt{s})$ and a_l^I , provide LECs for χ -PT
 - * obtain mass and decay width of resonance
- cons
 - ★ lattice artifacts arise from non-zero lattice spacing a
 - ★ finite volume constraints: no concepts of δ_l^I in a finite volume
- solutions:
 - ★ $N_f = 2$ maximally twisted fermions, automatically $O(a)$ improved
 - ★ Lüscher's finite size method: connecting finite volume to infinite volume

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4 Lüscher's method

- assume that $V^I(\vec{r})$ describe the interaction of $\pi\pi$ in the isospin channel, I .
- infinite volume: continuous energy spectrum \sqrt{s} :

$$V^I(\vec{r}) \Rightarrow \psi^I(\vec{r}) \Rightarrow \delta_l^I(\sqrt{s})$$

- lattice simulation ask for finite volume: discrete energy spectrum $\sqrt{s_L}$

$$V_L^I(\vec{r}) = \sum_{\vec{m} \in \mathbb{Z}^3} V(\vec{r} + \vec{m}L) \Rightarrow \sqrt{s_L}$$

$$V_L^I(\vec{r}) \Rightarrow \psi_L^I(\vec{r}) \Rightarrow (\delta_l^I)_L(\sqrt{s_L})??$$

- Lüscher established a connection between $\sqrt{s_L}$ and δ

$$\frac{1}{\tan \delta(\sqrt{s_L})} = Z(1; k_L L / (2\pi)) , \quad k_L = \sqrt{s_L/4 - m_\pi^2}$$

- $Z(1; k_L L / (2\pi))$ is a known function and universal for different interactions

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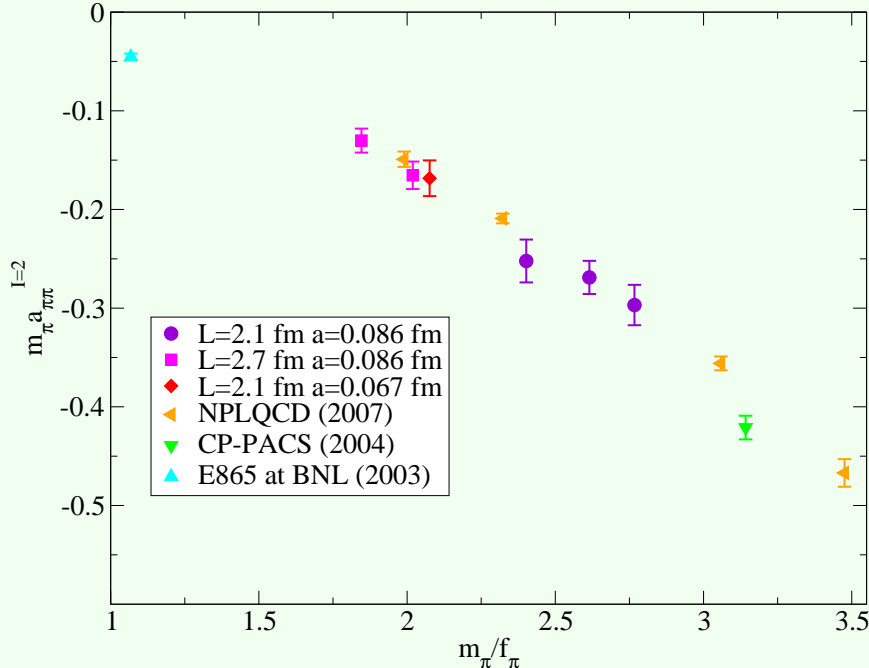
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5 $I=2$ channel

- twisted mass fermions:
 m_π : 270 MeV–480 MeV a : 0.086 fm, 0.067 fm
- calculate discrete energy spectrum $E_{\pi\pi}^{I=2}$ for each m_π
- Lüscher's method relates $E_{\pi\pi}^{I=2}$ to δ , and hence scattering length $a_{\pi\pi}^{I=2}$

$$E_{\pi\pi}^{I=2} - 2m_\pi = -\frac{4\pi a_{\pi\pi}^{I=2}}{m_\pi L^3} \left[1 + c_1 \frac{a_{\pi\pi}^{I=2}}{L} + c_2 \left(\frac{a_{\pi\pi}^{I=2}}{L} \right)^2 \right] + O(L^{-6})$$

where $c_1 = -2.837297$ and $c_2 = 6.375183$



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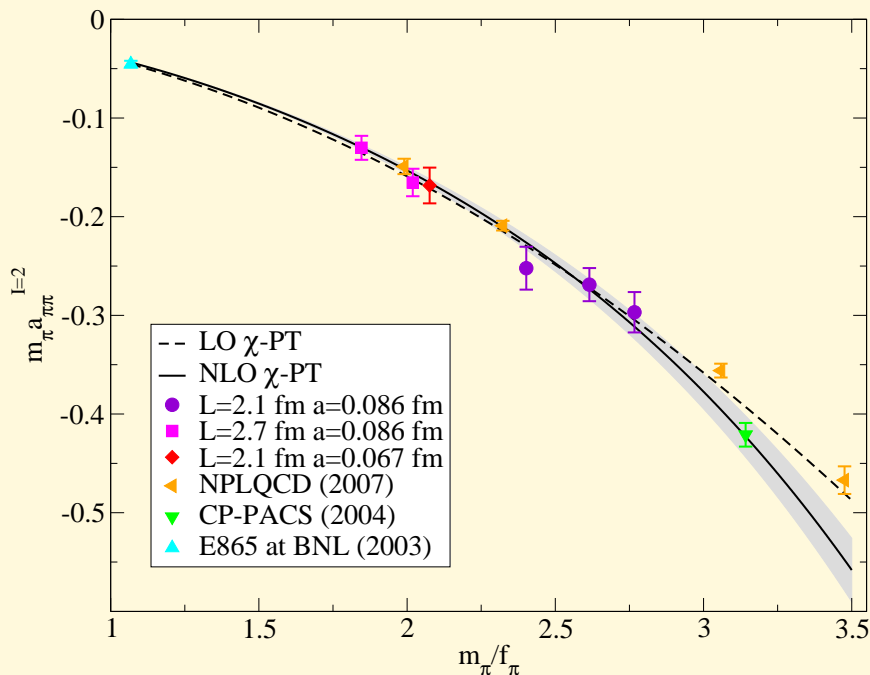
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• NLO χ -PT

$$m_\pi a_{\pi\pi}^{I=2} = -\frac{m_\pi^2}{8\pi f_\pi^2} \left\{ 1 + \frac{m_\pi^2}{16\pi^2 f_\pi^2} \left[3 \ln \left(\frac{m_\pi^2}{f_\pi^2} \right) - 1 - l_{\pi\pi}^{I=2}(\mu = f_{\pi,phy}) \right] \right\}$$



• this gives $m_\pi a_{\pi\pi}^{I=2}$ at physical limit and $l_{\pi\pi}^{I=2}(\mu)$ at a scale of $\mu = f_{\pi,phy}$

★ ETMC: $m_\pi a_{\pi\pi}^{I=2} = -0.04385(47)$ $l_{\pi\pi}^{I=2}(\mu) = 4.7(1.4)$

★ NPLQCD: $m_\pi a_{\pi\pi}^{I=2} = -0.04330(42)$ $l_{\pi\pi}^{I=2}(\mu) = 6.2(1.2)$

★ CGL: $m_\pi a_{\pi\pi}^{I=2} = -0.0444(10)$

★ E865 at BNL: $m_\pi a_{\pi\pi}^{I=2} = -0.0454(34)$



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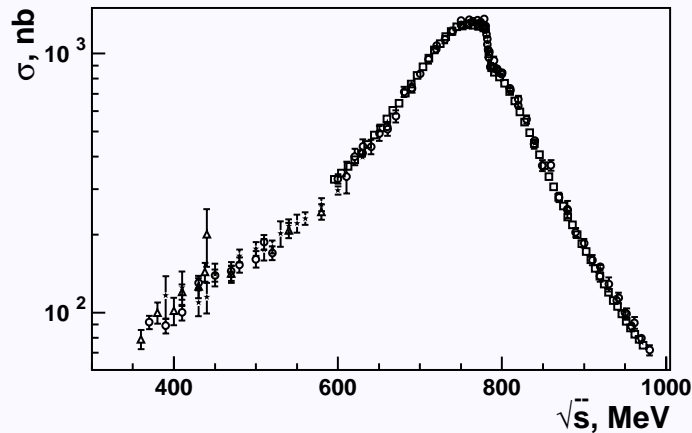
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6 $l=1$ channel

- resonance $\rho(770)$ in $e^+e^- \rightarrow \pi^+\pi^-$ cross section, large width: $\Gamma=149$ MeV:



- cross section σ is contributed by l -th partial wave scattering phase δ_l^I :

$$\sigma(\sqrt{s}) \propto \sum_l (2l + 1) \sin^2(\delta_l^I(\sqrt{s}))$$

- $I = 1$ channel, P-wave scattering phase δ_1^1 dominates the contribution to σ
- definition for resonance mass M_R :

$$\sigma(\sqrt{s}) \Big|_{\sqrt{s}=M_R} = \sigma_{\max} \quad \text{or} \quad \delta_1^1(\sqrt{s}) \Big|_{\sqrt{s}=M_R} = \frac{\pi}{2}$$



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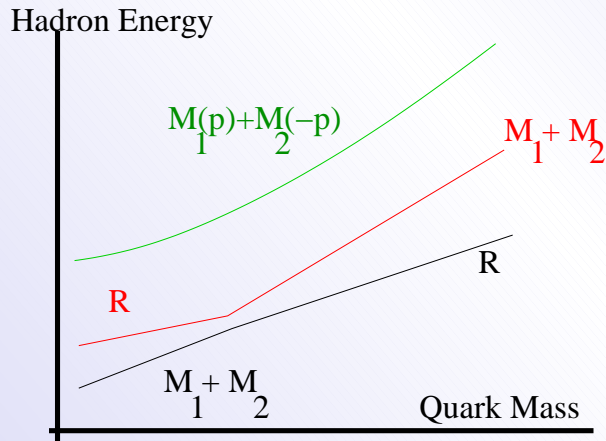
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- twisted mass fermions:

$$m_\pi = 390, 310, 270 \text{ MeV}, \quad L/a = 24, 32, 32, \quad a = 0.086 \text{ fm}$$



- $m_{\rho^0} > 2m_{\pi^+}$: threshold is open for $\rho^0 \rightarrow \pi^+\pi^-$
- elastic scattering region: $2m_\pi < \sqrt{s_L} < 4m_\pi$
- collect four (three) energy spectrums $E_{\pi\pi}^{I=1}$ for each m_π
- insert $\sqrt{s_L} = E_{\pi\pi}^{I=1}$ into Lüscher formula, evaluate scattering phase $\delta_1^1(\sqrt{s_L})$

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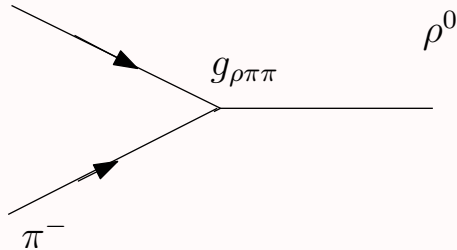


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- call effective range formula to describe the resonant behavior of δ_1^1

$$\tan \delta_1^1(k) = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{k^3}{\sqrt{s}(M_R^2 - s)}, \quad k = \sqrt{s/4 - m_\pi^2}$$

- determine resonance mass M_R and coupling constant $g_{\rho\pi\pi}$



- decay width is given by $g_{\rho\pi\pi}$:

$$\Gamma = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{k^3}{M_R^2}, \quad k = \sqrt{M_R^2/4 - m_\pi^2}$$

- NLO χ -PT theory predicts: $g_{\rho\pi\pi}$ is almost m_π independent

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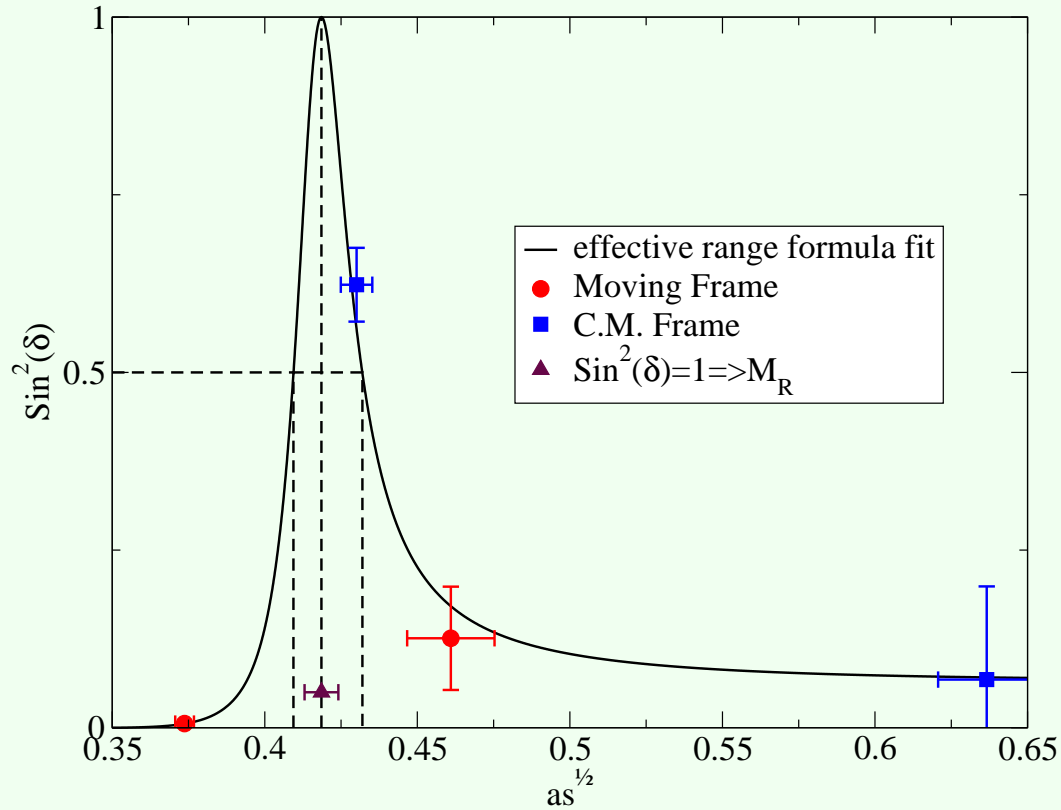
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• $m_{\pi^+} = 390 \text{ MeV}$



• $aM_R = 0.4186(56)$, $g_{\rho\pi\pi} = 6.16(48)$

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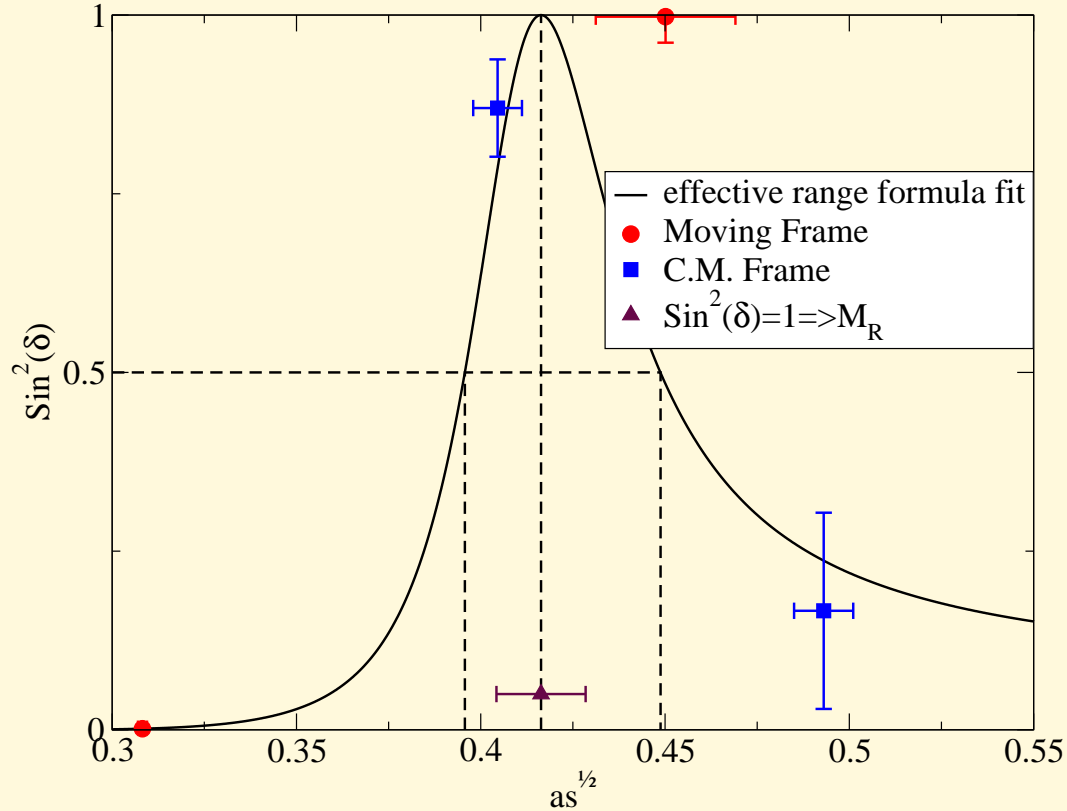
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• $m_{\pi^+} = 310 \text{ MeV}$



• $aM_R = 0.416(12)$, $g_{\rho\pi\pi} = 6.33(84)$

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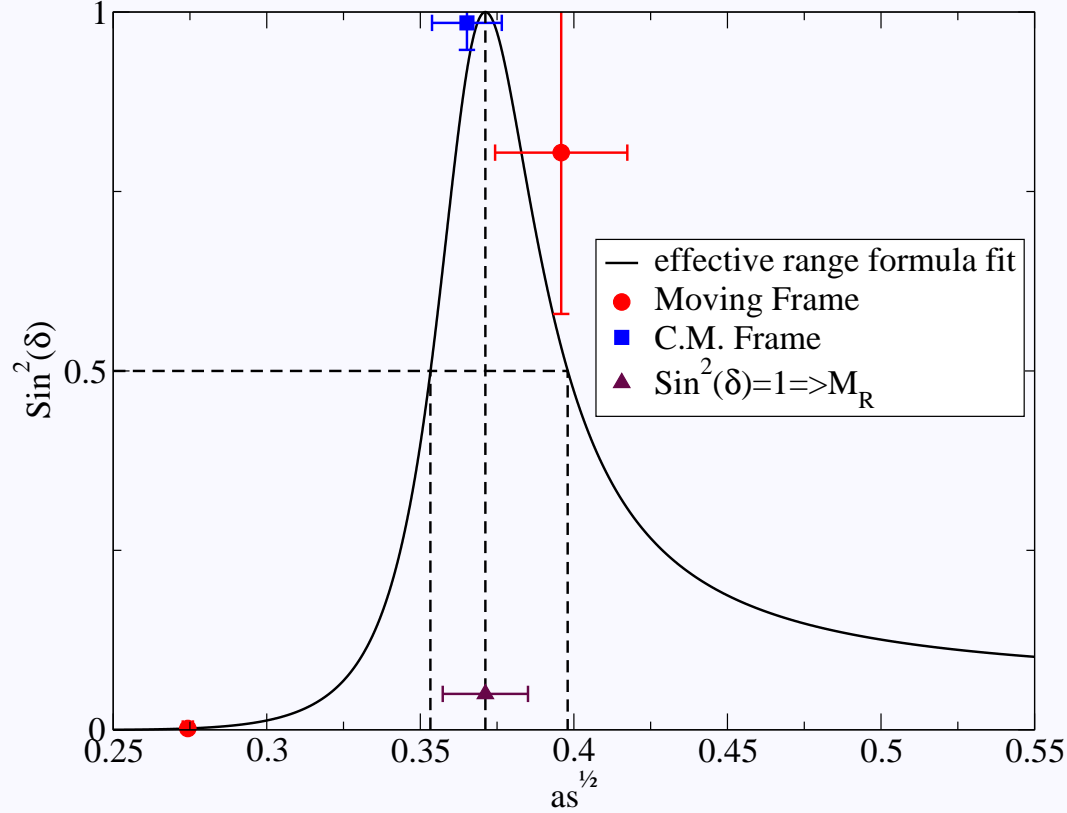
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• $m_{\pi^+} = 270$ MeV



• $aM_R = 0.371(14)$, $g_{\rho\pi\pi} = 6.04(77)$

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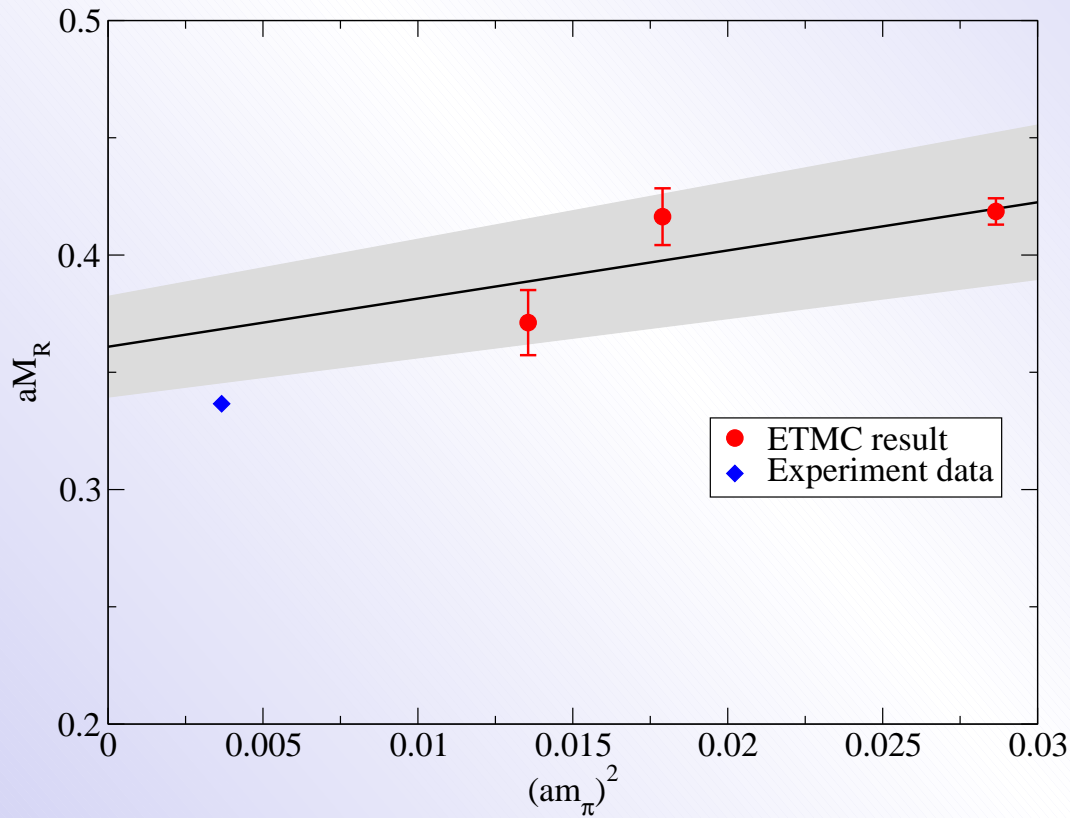
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- chiral extrapolation for M_R

$$M_R = M_R^0 + c_1 m_\pi^2 + O(m_\pi^3)$$



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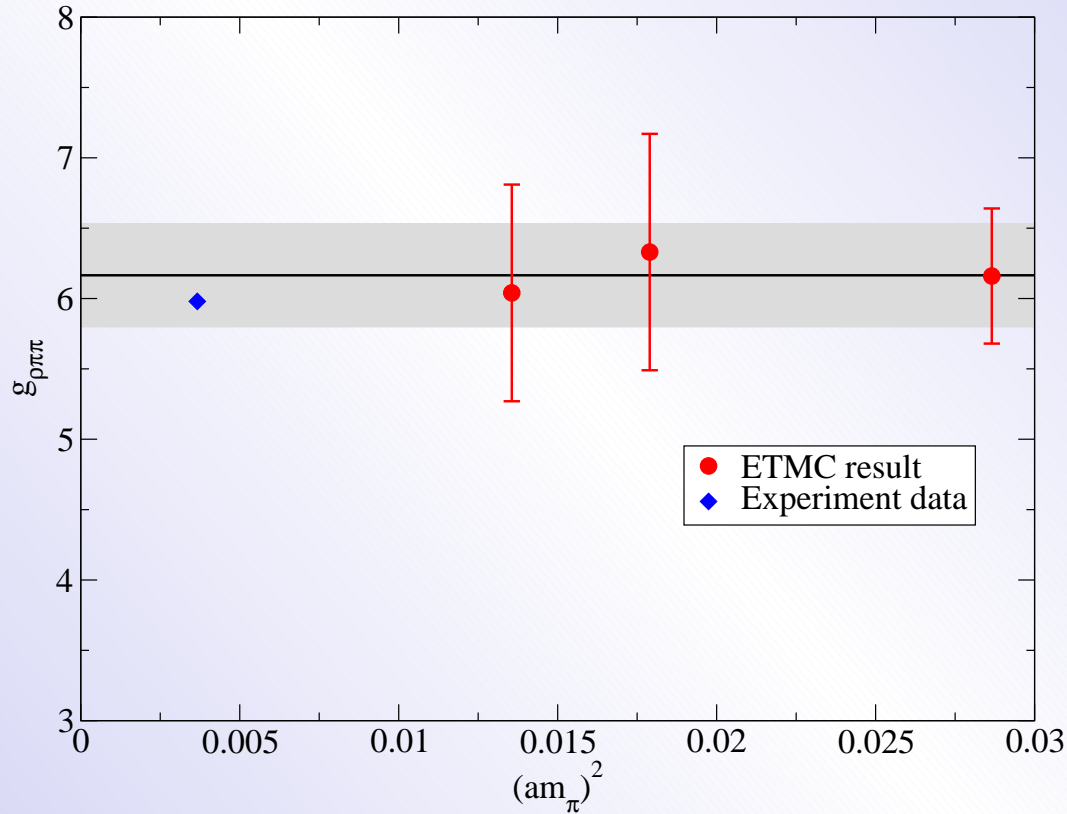
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- fit $g_{\rho\pi\pi}$ as a constant



- ETMC: $g_{\rho\pi\pi} = 6.17(37)$ Experiment: $g_{\rho\pi\pi} = 5.98(02)$

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7 Conclusion

- in $I = 2$ channel, we probe the chiral dynamics of strong interaction
 - ★ smaller m_π is used: 270 MeV–480 MeV
 - ★ precise scattering length at physical limit:
 - ETMC: $m_\pi a_{\pi\pi}^{I=2} = -0.04385(47)$
 - NPLQCD: $m_\pi a_{\pi\pi}^{I=2} = -0.04330(42)$
 - E865 at BNL: $m_\pi a_{\pi\pi}^{I=2} = -0.0454(34)$
- in $I = 1$ channel
 - ★ at all three pion masses, we get obvious indications of resonance
 - * $m_\pi = 390$ MeV , $aM_R = 0.419(06)$, $g_{\rho\pi\pi} = 6.16(48)$
 - * $m_\pi = 310$ MeV , $aM_R = 0.416(12)$, $g_{\rho\pi\pi} = 6.33(84)$
 - * $m_\pi = 270$ MeV , $aM_R = 0.371(14)$, $g_{\rho\pi\pi} = 6.04(77)$
 - ★ chiral extrapolation for M_R and $g_{\rho\pi\pi}$
 - * M_R : deviation from experiment is a little more than 1σ
only three data, linear fit??
 - * $g_{\rho\pi\pi}$: well consistent with experiment
pion mass dependence of $g_{\rho\pi\pi}$ is small
- future improvement:
 - ★ calculate M_R and $g_{\rho\pi\pi}$ at smaller lattice spacing to check lattice artifacts
 - ★ extract scattering parameter in $I = 0$ channel



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