

Neutrino Pendulum

Part I: General Introduction

Part II: 3-flavor Neutrino Mixing

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Graduate School Rathen

Part I: Coupled Pendula

- Free Oscillation of one pendulum: $\omega_0^2 = \frac{g}{\ell}$
- 2 pendula with same length ℓ , mass m coupled by spring with strength k
- 2 Eigenmodes

– Different eigenfrequencies = energies

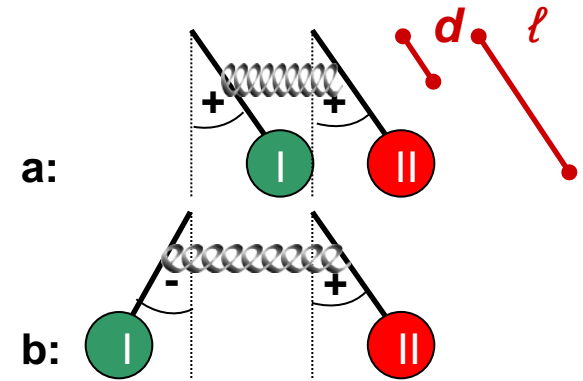
Mode a (II + I) with $\omega_a^2 = \omega_0^2$

Mode b (II - I) with $\omega_b^2 = \omega_0^2 + \Delta\omega^2$

– **Frequency (=energy) difference increases with stronger coupling**

$$\Delta\omega^2 = \frac{kd^2}{m\ell^2}$$

– Coupling can be steered by varying k or d (we'll vary d in the following)



Two bases in Hilbert-space

flavor-basis

- eigenstates of flavor
- eigenstates of weak charge
- *particles take part in weak interactions as flavor-eigenstates*

- Examples:

- $\bar{K}^0(\bar{s} u)$ or $K^0(\bar{s} u)$
- ν_e, ν_μ, ν_τ

mass-basis

- eigenstates of mass
- well-defined lifetime
- *Particles propagate through space-time as mass-eigenstates*

$$|\nu(\mathbf{t})\rangle = |\nu\rangle \mathbf{e}^{i(\vec{p}\vec{x} - Et)} \mathbf{e}^{-\Gamma t}$$

- Examples:

- K_L^0, K_S^0
- ν_1, ν_2, ν_3

- Like coupled pendula, the coupling of particles leads to eigenstates with different masses and lifetimes, e.g. for linear combination of 2 states:

$$\nu_a = (\nu_\tau + \nu_\mu)/\sqrt{2} \quad \text{with} \quad m_a^2 = m_0^2$$

$$\nu_b = (\nu_\tau - \nu_\mu)/\sqrt{2} \quad \text{with} \quad m_b^2 = m_0^2 + \Delta m^2$$

Correspondences

pendulum	particles
Linear oscillation	complex phase rotation
Eigenmodes → fixed eigenfrequencies	Mass eigenstates → fixed phase frequencies
Frequency differences $\Delta\omega$ → different energies	Frequency differences $e^{i\Delta Et} \sim e^{i\Delta m^2 t}$ → different masses
One pendulum = lin. combination of eigenmodes	Flavor eigenstate = lin. combination of mass eigenstates
$ \text{amplitude}^2 \sim$ total energy in oscillation	$ \text{amplitude}^2 \sim$ detection probability
Beat-Frequency $\sim \Delta\omega$ of eigenmodes	Flavor-Oscillation $\sim \Delta m^2$ of mass eigenstates

Part II: Neutrino flavor pendulum



**coupled pendula
for demonstrating
3-flavor neutrino
mixing as realized in
nature**

Idea: M.K.

built 2004 at Uni Bonn,
extended 2006 at TU Dresden
with variable mixing angles
and digital readout

<http://neutrinopendel.tu-dresden.de>



Copies in: Hamburg, Münster, DESY(Zeuthen), ...

3-flavor neutrino mixing



$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

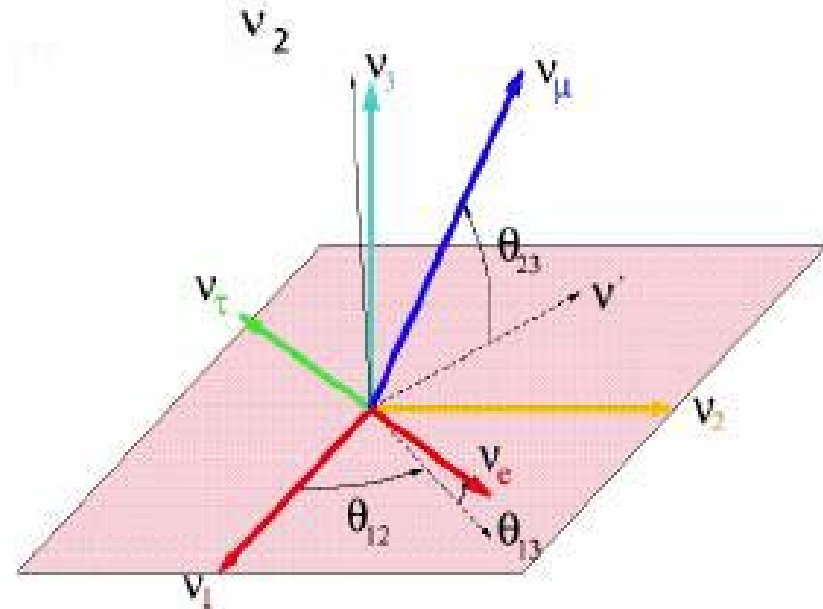
$\Theta_{\text{atmos, beam}}$

θ_{13}, δ

$\Theta_{\text{solar, reactor}}$

PMNS mixing matrix
(w/o Majorana Phases)

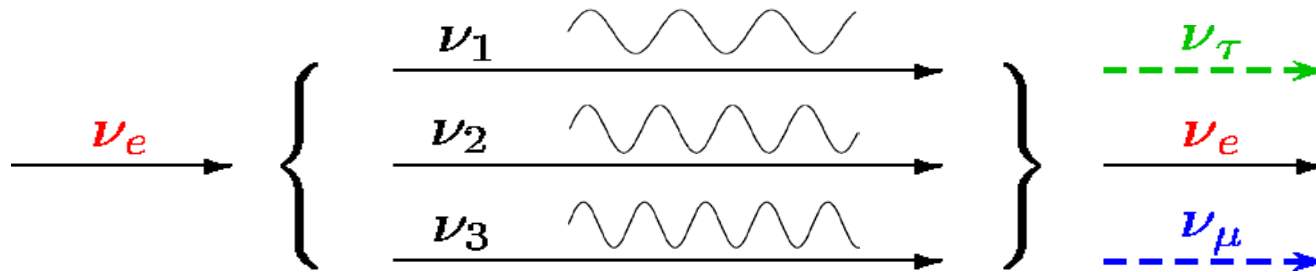
- 3 Mixing angles: $\theta_{12}, \theta_{23}, \theta_{13}$
- 1 CP-violating Dirac-Phase: δ
(neglected in the following)



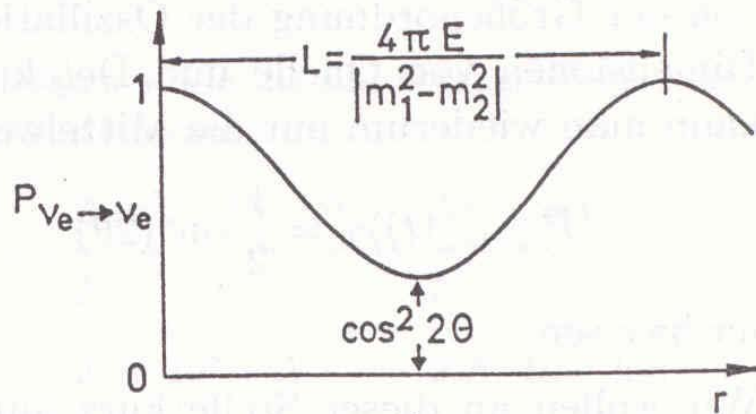


ν flavor-oscillations

- Each flavor (e.g. ν_e) is sum of mass eigenstates (ν_1, ν_2, ν_3)
- Each mass eigenstate with fixed p has a different phase frequency ω_i
 - $\exp(i\omega_i t) = \exp(iE_i t) = \exp(i(\sqrt{p^2 + m_i^2})t) \sim \exp(ipt + im_i^2 t/2p + \dots)$



- The differences $\Delta\omega_{ij} \sim |m_i^2 - m_j^2| =: \Delta m_{ij}^2$ lead to flavor oscillations
 - Δm_{ij}^2 determines the oscillation **period**
 - θ_{ij} determines the oscillation **amplitude**



$$L_{ij} = 2.5m \frac{E(\text{MeV})}{\Delta m_{ij}^2(\text{eV}^2)}$$



Current values

cf. global fit Th.Schwetz et al., NJP 10 (2008)

$\Delta m^2_{23} = 2,4 \times 10^{-3} \text{ eV}^2$	$\Delta m^2_{13} = 2,5 \times 10^{-3} \text{ eV}^2$	$\Delta m^2_{12} = 0,08 \times 10^{-3} \text{ eV}^2$
„fast“ oscillation		„slow“ oscillation
$L_{23} = 1 \text{ km} \times E(\text{MeV})$		$L_{12} = 30 \text{ km} \times E(\text{MeV})$
$\theta_{23} = 45^\circ \pm 3^\circ$	$\theta_{13} < 11^\circ$ (90% CL)	$\theta_{12} = 33.5^\circ \pm 1.5^\circ$

$\theta_{\text{atmos, beam}}$

θ_{13}, δ

$\theta_{\text{solar, reactor}}$

- consistent with so-called tri/bi-maximal mixing

$$\theta_{23} = 45^\circ$$

$$\theta_{13} = 0^\circ$$

$$\theta_{12} = 35.3^\circ$$

$$U_{\text{PMNS}} \approx \begin{pmatrix} \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Harrison, Perkins, Scott '99,'02
 Z.Xing,'02, He, Zee, '03, Koide '03
 Chang, Kang, Kim '04, Kang '04

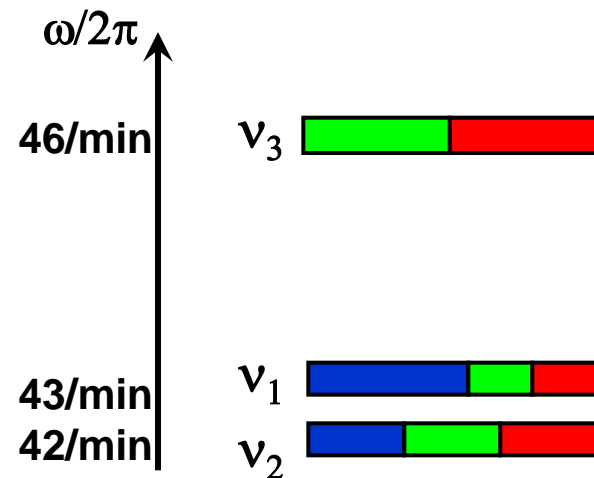
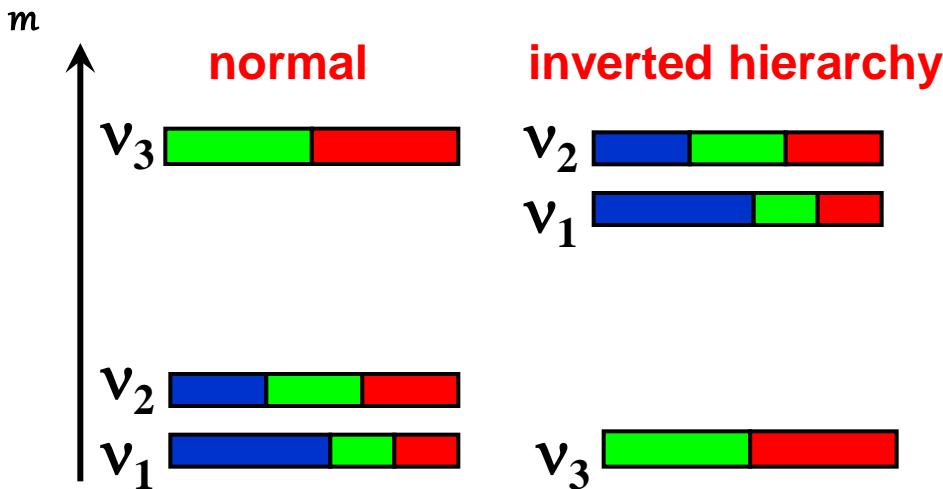
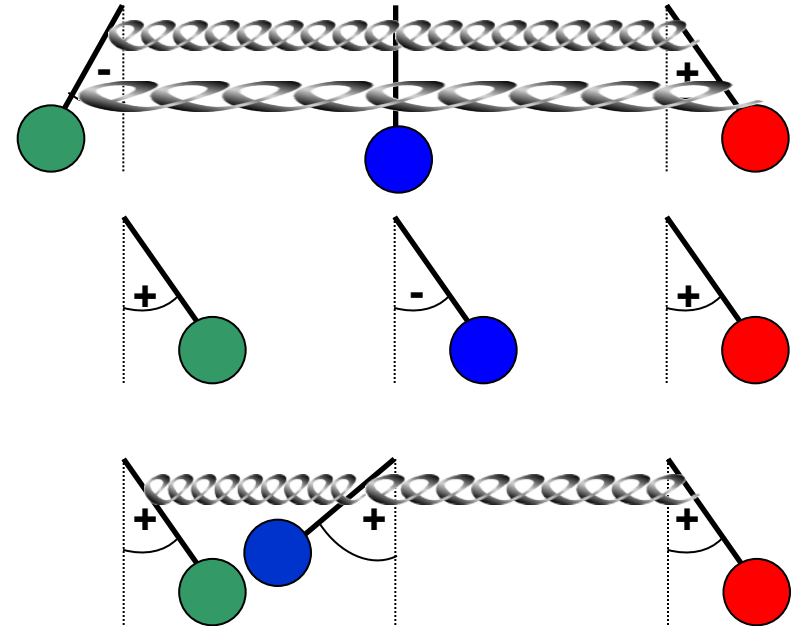
Realisation as coupled pendula



$$\nu_3 = (-\nu_\mu + \nu_\tau)/\sqrt{2}$$

$$\nu_2 = (-\nu_e + \nu_\mu + \nu_\tau)/\sqrt{3}$$

$$\nu_1 = (2\nu_e + \nu_\mu + \nu_\tau)/\sqrt{6}$$



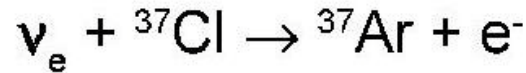
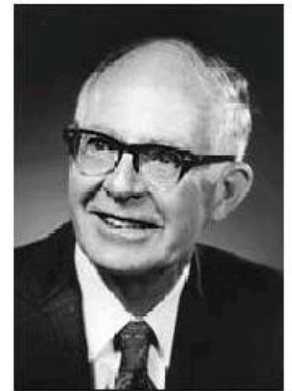
The solar neutrino „deficit“



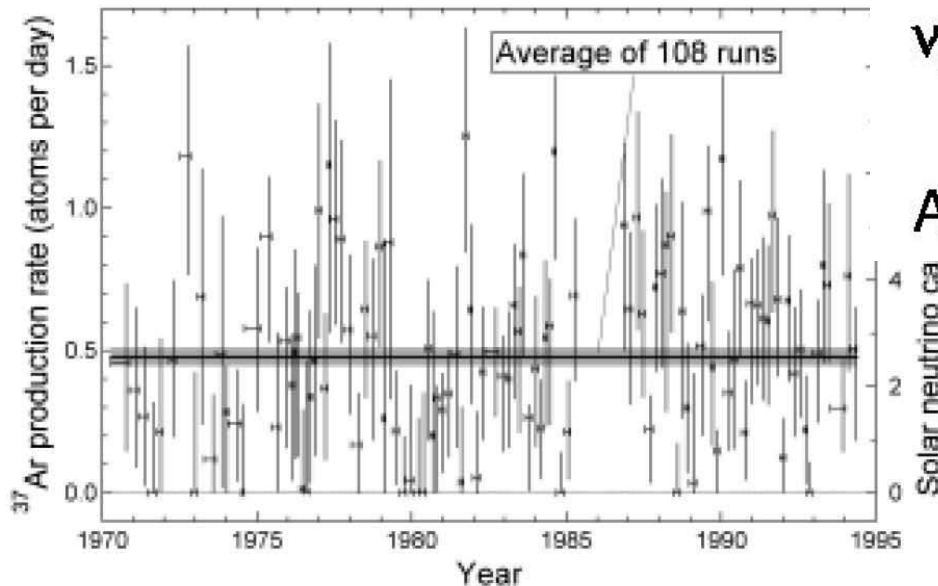
Ray Davis

Nobelpreis 2002

380000 l
Perchlorethylen
in der Homestake- Mine



Ausspülen des ${}^{37}\text{Ar}$ (0.5 Atome/Tag)



- Davis: only sensitive to ν_e
Result: Only 32% of expected ν_e detected

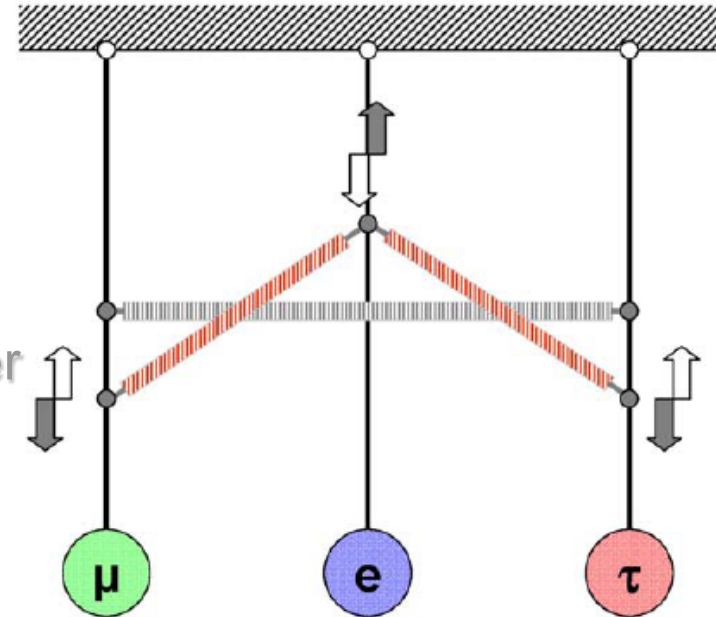
Modify θ_{12}



- Modify fraction of ν_e in ν_1 and ν_2
- $\nu_2 = (-\nu_e + \nu_\mu + \nu_\tau)/\sqrt{3}$ no longer eigenmode

Possible range:
 $20^\circ < \theta_{12} < 90^\circ$

θ_{12} smaller
 θ_{12} larger



- <http://neutrinoependel.tu-dresden.de>
 (special high school thesis J. Pausch 2008)

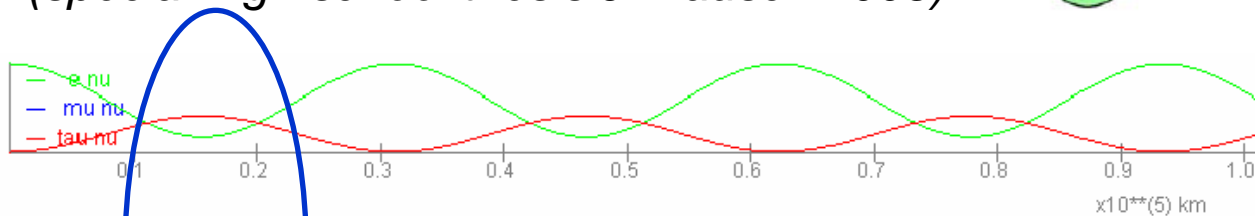


Abbildung 27: Sonnenneutrino-Oszillation mit $\theta_{12}=0,57\text{rad}$ (33°), Java-Applet.

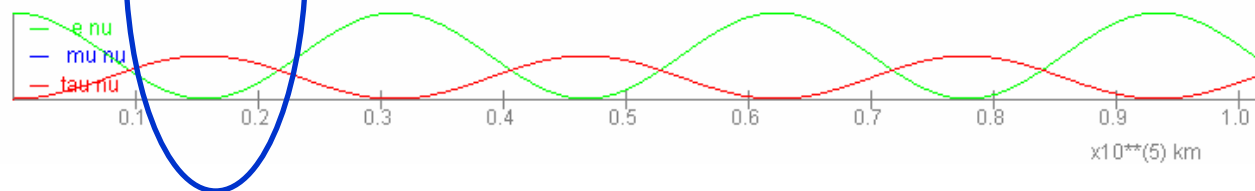


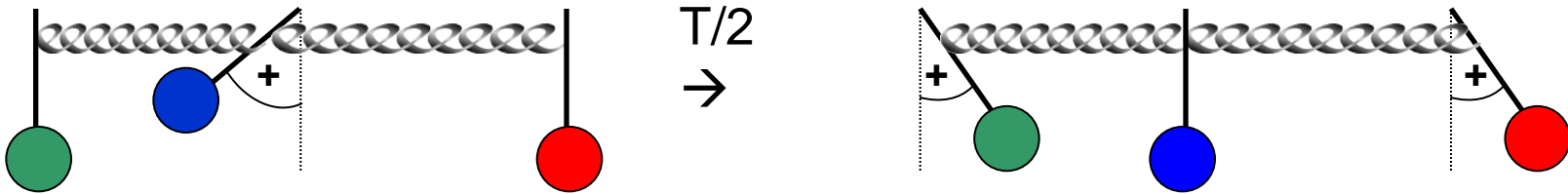
Abbildung 28: Sonnenneutrino-Oszillation mit $\theta_{12}=0,7854\text{rad}$ (45°), Java-Applet.

Need for enhancement (MSW effect)

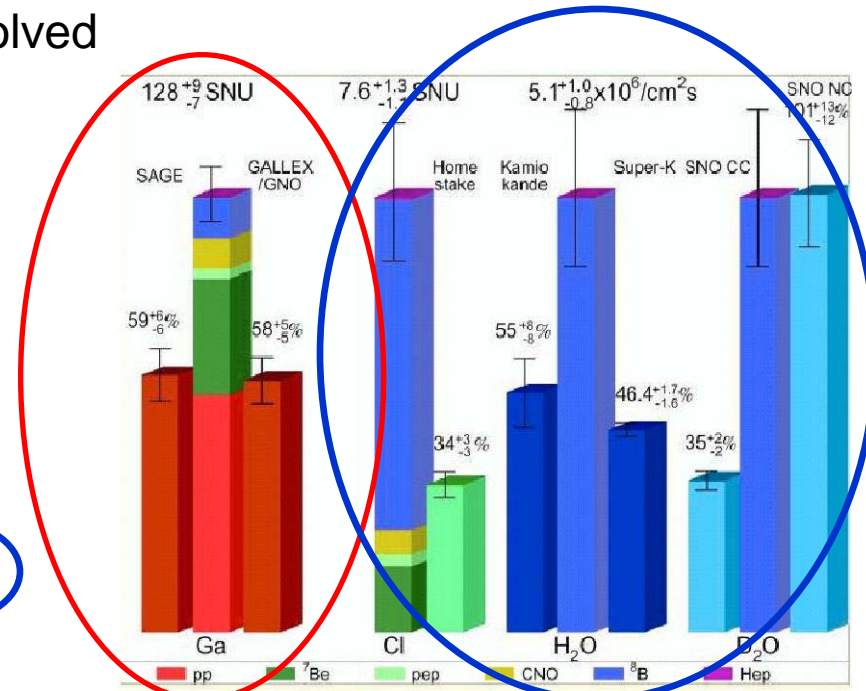
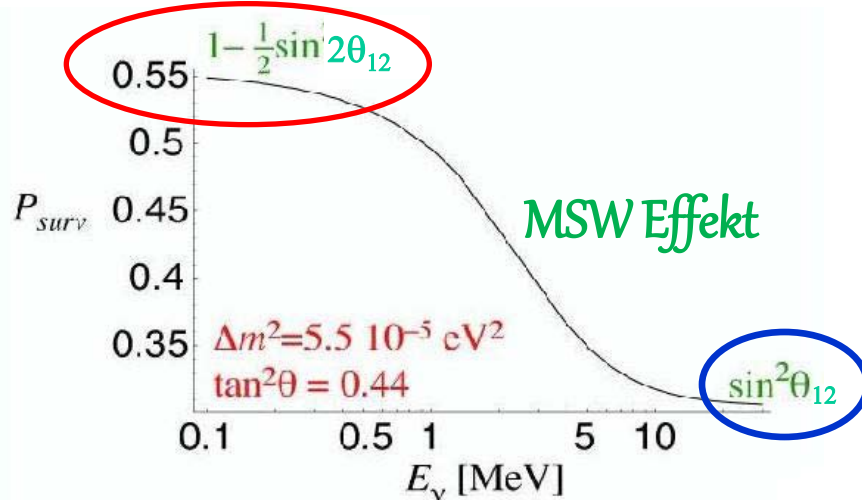


- nuclear fusion: 100% ν_e leave the sun (w/o MSW effect)
 $4p \rightarrow {}^4\text{He} + 2e^+ + 2\nu_e + 27 \text{ MeV}$

- “slow” oscillation via θ_{12} and small Δm^2_{12} (pendula: weak coupling)



- oscillation only to $(\nu_\tau + \nu_\mu)/\sqrt{2}$
- transition to $(\nu_\tau - \nu_\mu)/\sqrt{2}$ not possible, since ν_e not in \mathbf{V}_3
- $P(\nu_e \rightarrow \nu_e) > 50\%$ since just \mathbf{V}_1 and \mathbf{V}_2 involved
 \rightarrow need for enhancement (MSW effect)

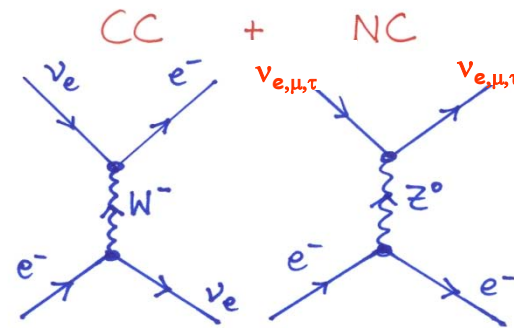


Problems solved 1985 by MSW (Mikheyev–Smirnov–Wolfenstein) effect



- Historical Prejudice: mixing angle should be small
 - **Problem: How to get large neutrino deficit w/ small mixing?**
 - Today no problem: 2 mixing angles are large!
- Knowing about large θ_{12} , but having $\theta_{13} = 0$
 - Effective 2-flavor mixing!
→ min detection rate should be $\geq 50\%$
 - **Problem: Observed rate of Homestake $\sim 32\%$!**

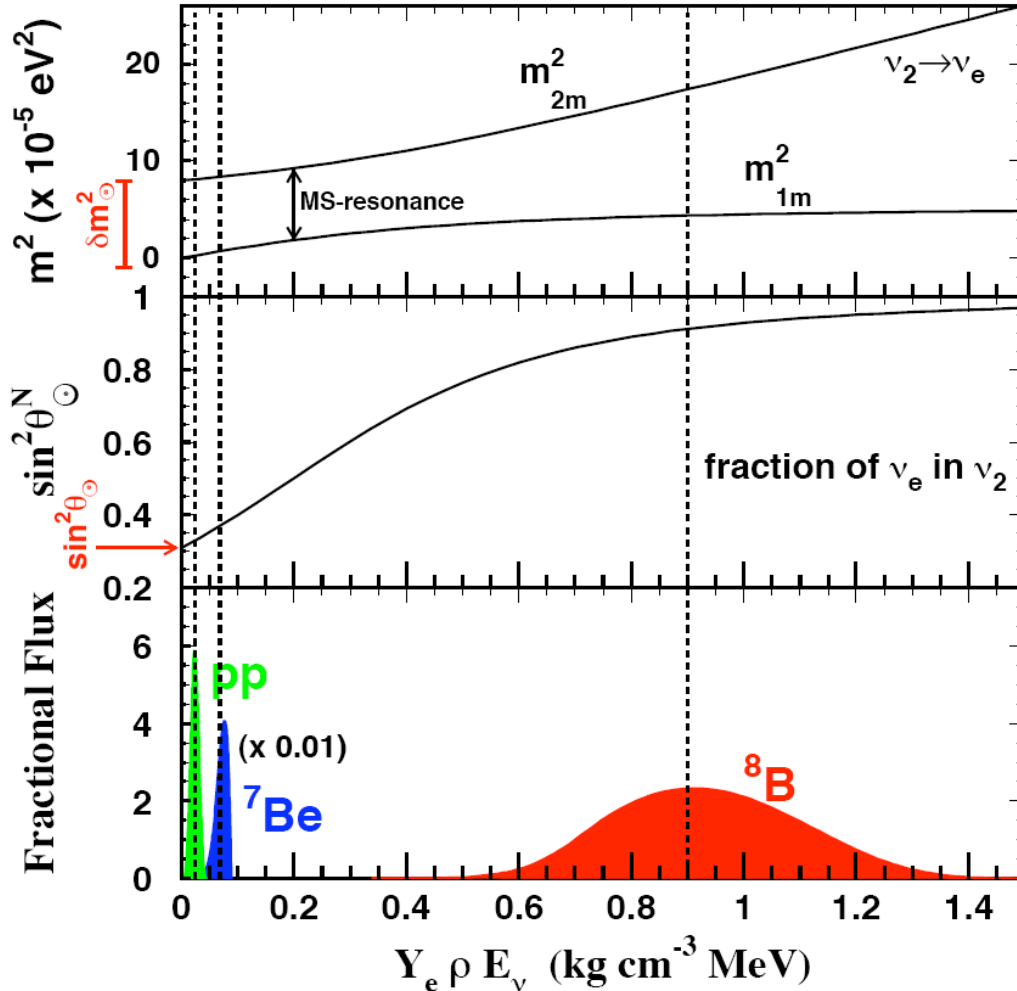
In matter there is an **additional potential** in the equation of motion for $\nu_e \rightarrow \nu_e$ scattering:



$$\text{Matter: } i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m^2 \cos 2\theta + 2\sqrt{2}G_F N_e E & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & \Delta m^2 \cos 2\theta - 2\sqrt{2}G_F N_e E \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$\text{with } 2\sqrt{2}G_F \underbrace{N_e}_{\frac{Y_e \rho}{m_p}} E = 1.53 \cdot 10^{-7} \text{ eV}^2 \left(\frac{Y_e \rho}{\text{g/cm}^3} \cdot \frac{E}{\text{MeV}} \right) \quad \text{center of Sun: } \frac{Y_e \rho}{\text{g/cm}^3} \cong 100$$

Today's values for Solar ν



In Vacuum

$$\delta m_{\odot}^2 = 8.0 \pm 0.4 \times 10^{-5} \text{ eV}^2$$

$$\sin^2 \theta_{\odot} = 0.31 \pm 0.03$$

Whereas for ${}^8\text{B}$
at center of Sun

$$\delta m_N^2 = 14 \times 10^{-5} \text{ eV}^2$$

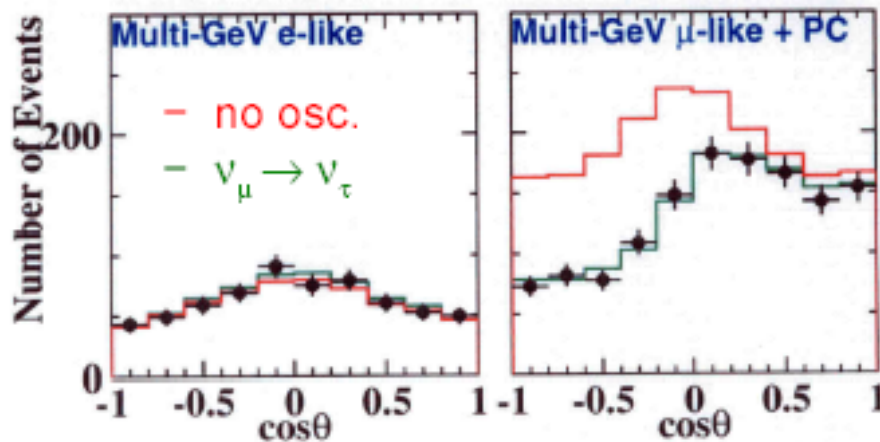
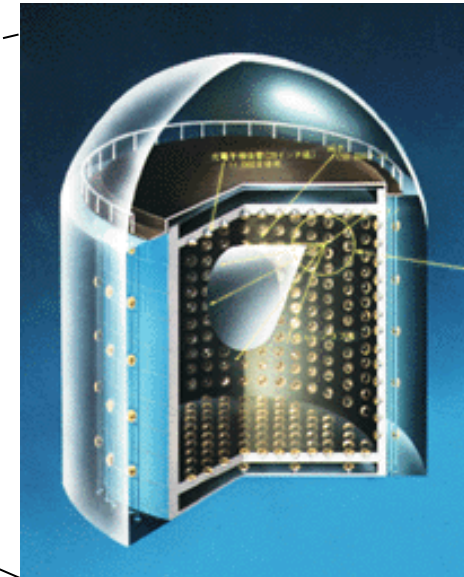
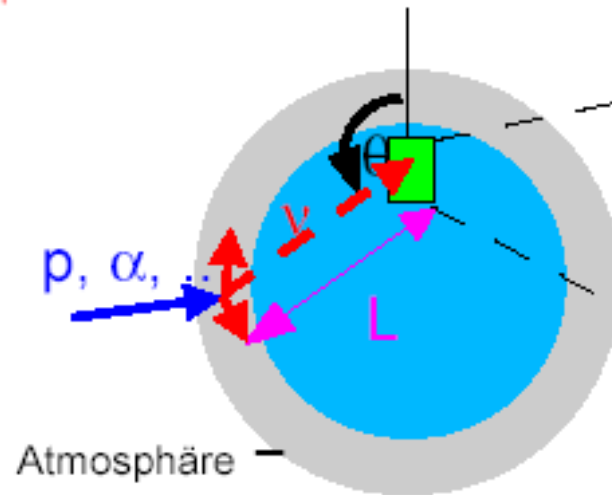
$$\sin^2 \theta_{\odot}^N = 0.91$$

Atmospheric neutrinos



$\bar{\nu}_\mu$ und $\bar{\nu}_e$ aus π/K -Zerfällen

SuperKamiokande 2000:



(C. Mc Grew, NOON 2000, Dez.2000)

look at ν_e and ν_μ from air showers:

- no deficit for ν_e
- clear deficit for ν_μ
- fully compatible with $\nu_\mu \rightarrow \nu_\tau$

atmospheric neutrinos



- SuperKamiokande 2000:

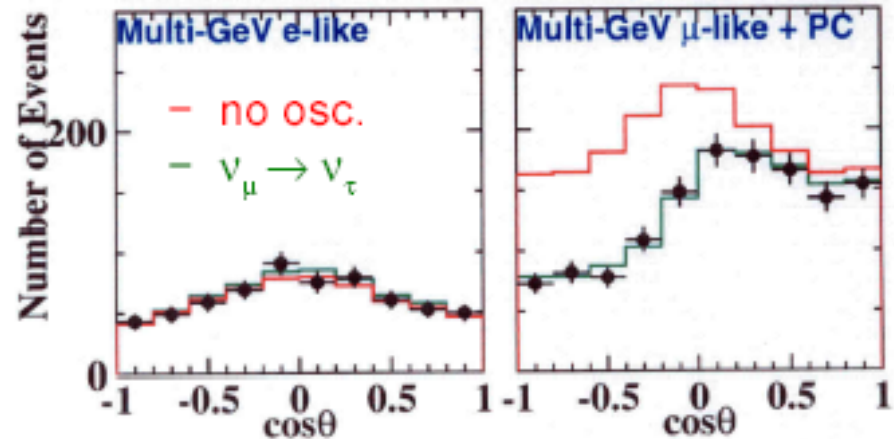
described als $\nu_{\mu} \rightarrow \nu_{\tau}$

- pendula:

ν_e : weak coupling to ν_{μ}, ν_{τ}

ν_{μ} : weak coupling to ν_e

strong coupling to ν_{τ}

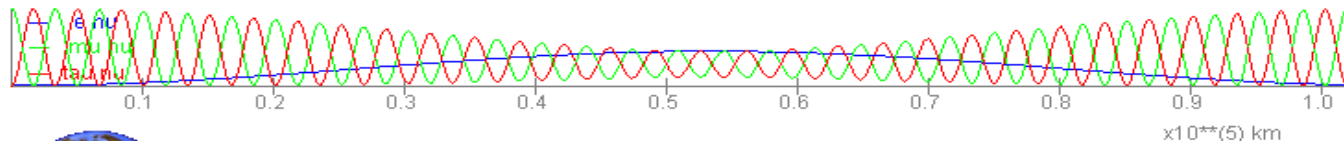


Interactive Neutrino Oscillation Laboratory

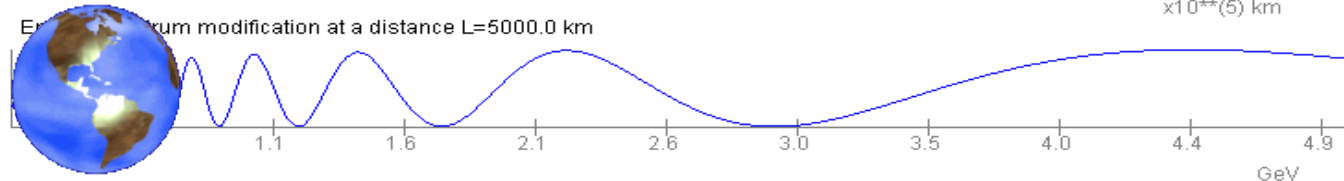
Three Generations Neutrino Oscillations

Adam Para, Fermilab

Appearance/disappearance probability as a function of distance, for $E_{\nu} = 3.0$ GeV



Energy spectrum modification at a distance $L=5000.0$ km



1 = 0.166

2 = 0.333

3 = 0.500

composition of the initial neutrino in terms of mass eigenstates



Mixing Matrix

0.816	0.577	0.0
-0.40	0.577	0.707
0.408	-0.57	0.707

1

2

3

e
μ
τ



e = 0.009

μ = 0

τ = 0.990

composition of the 3.0 GeV flux at 5000 km in terms of flavor states

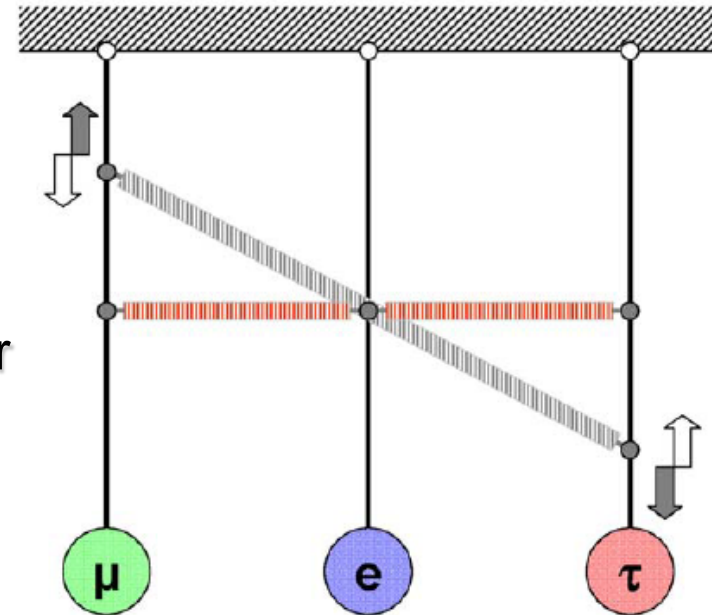
Modify θ_{23}



- Non-maximal mixing of ν_μ and ν_τ
- $\nu_3 = (-\nu_\mu + \nu_\tau)/\sqrt{2}$ no longer eigenmode

Possible range: $30^\circ < \theta_{23} < 60^\circ$

θ_{23} smaller
 θ_{23} larger



- <http://neutrinoependel.tu-dresden.de>
 (special high school thesis J. Pausch 2008)

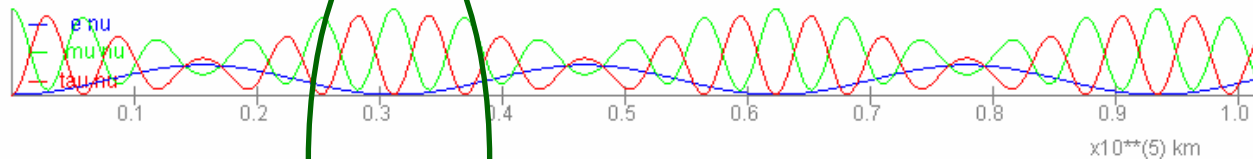


Abbildung 52: Atmosphärische Neutrino-Oszillation mit $\theta_{23} = 0,9\text{rad}$ ($51,6^\circ$), Java-Applet.

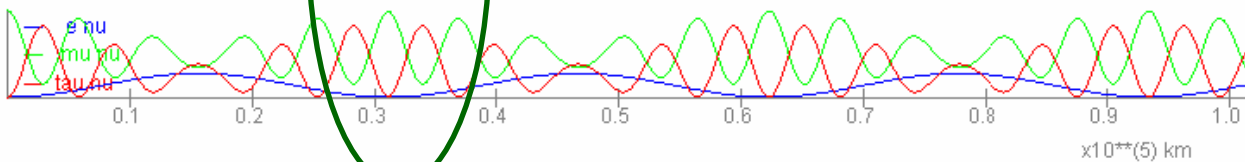
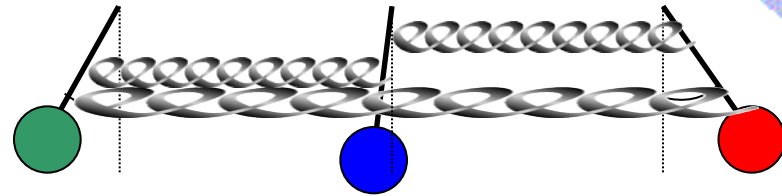


Abbildung 53: Atmosphärische Neutrino-Oszillation mit $\theta_{23} = 0,98\text{rad}$ ($56,2^\circ$), Java-Applet.

Impact of θ_{13}



- $\nu_3 = (\sin\theta_{13}\nu_e - \nu_\mu + \nu_\tau)/\sqrt{2.01}$

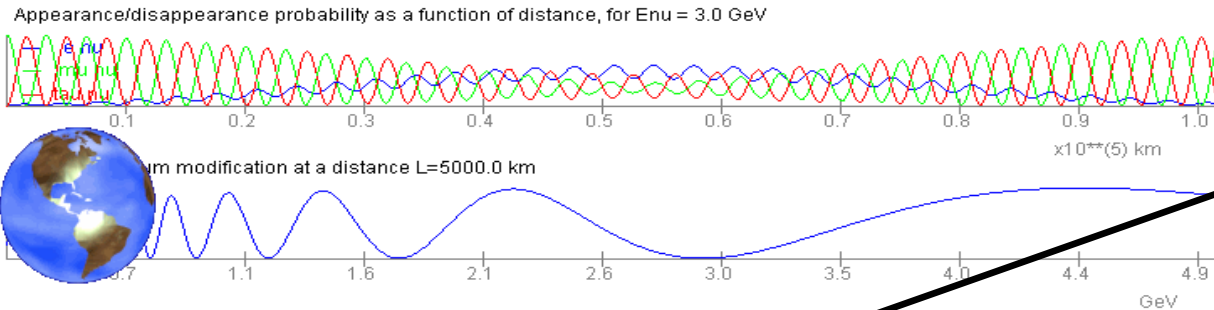


- reactor $\bar{\nu}_e \rightarrow \bar{\nu}_\tau + \bar{\nu}_\mu$ disappearance and atmospheric or beam $\nu_\mu \rightarrow \nu_e$ appearance

- „slow“ directly via Δm_{12} (weak coupling)
- „fast“ modulation via $\nu_\tau - \nu_\mu$ with Δm_{23} (strong coupling)

Interactive Neutrino Oscillation Laboratory

Three Generations Neutrino Oscillations Adam Para, Fermilab



1 = 0.217
2 = 0.287
3 = 0.495

composition of the initial neutrino in terms of mass eigenstates

Mixing Matrix

0.812	0.574	0.099
-0.46	0.536	0.703
0.350	-0.61	0.703
1	2	3

e = 0.029
mu = 0
tau = 0.970

composition of the 3.0 GeV flux at 5000. km in terms of flavor states

$$\theta_{13} = 6^\circ$$

$$\sin \theta_{13} = 0.1$$

$$\sin^2 2\theta_{13} = 0.04$$

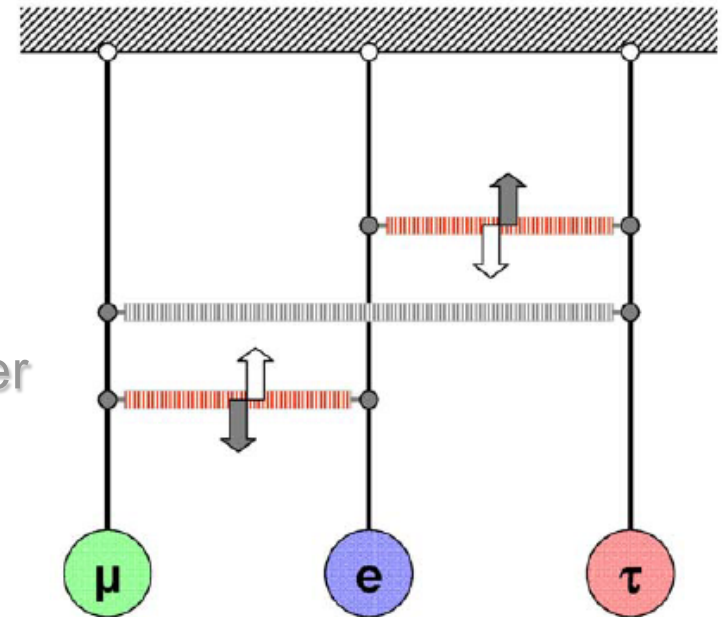
Modify θ_{13}



- ν_e present in $\nu_3 \sim (\sin \theta_{13} \nu_e - \nu_\mu + \nu_\tau)$
- ν_e can now excite $(\nu_\tau - \nu_\mu)$ mode, inducing fast $\nu_\tau - \nu_\mu$ modulation

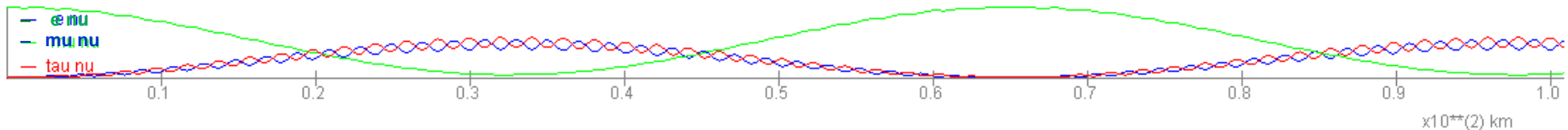
Possible range:
 $-6^\circ < \theta_{13} < 6^\circ$

θ_{13} smaller
 θ_{13} larger



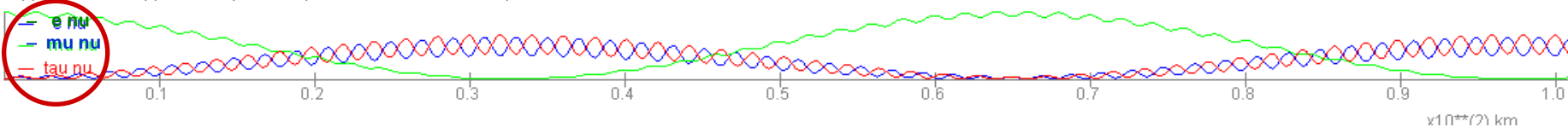
- Reactor neutrinos (2 MeV)
 - $\sin \theta_{13} = 0.10$ ($\theta_{13} = 6^\circ$)

Appearance/disappearance probability as a function of distance, for $E_{\nu} = 0.0020$ GeV



- $\sin \theta_{13} = 0.20$ ($\theta_{13} = 12^\circ$)

Appearance/disappearance probability as a function of distance, for $E_{\nu} = 0.0020$ GeV



Are neutrino pendula a perfect model?



- Few “features”
 - Need “creative” sign convention, leading to
 - imperfection for understanding sequence of masses
 - imperfection for $\theta_{23} \neq 45^\circ$
 - ◆ some $(\mathbf{v}_\tau - \mathbf{v}_\mu)$ present in \mathbf{v}_1 and \mathbf{v}_2
 - ◆ but $\mathbf{v}_e \rightarrow (\mathbf{v}_\tau - \mathbf{v}_\mu)$ still not possible!
- Else perfect!

The END !