

On the Gluon Propagator at Finite Temperature Lattice QCD

SU(3) Pure Gauge Investigation

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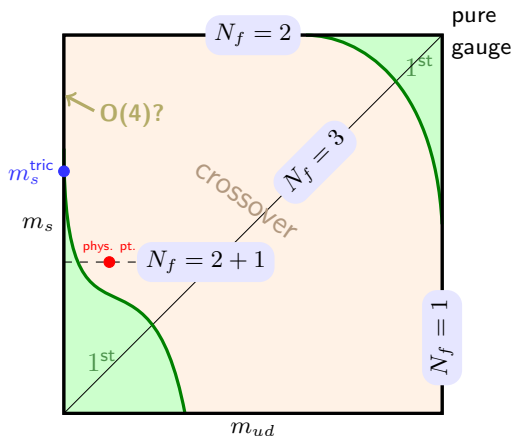
2 Results

3 Summary and Outlook

Why QCD at Finite Temperature?

- The early universe under extreme conditions (density and temperature)
- Quark gluon plasma is produced at high energies in heavy ion collisions
- The equation of state
- Identify the nature of the phase transition and establish the phase diagram

Nature of the Phase Transition of QCD



- Investigate the thermal transition by means of LQCD
- Here: $N_f = 0$ and vanishing chemical potential
- Expect a first order transition

Motivation of this work

- Tiny signal of the crossover \implies intensive computational efforts to reach the realistic case (physical quark masses)
- The need of more observables sensitive to the phase transitions using LQCD
- Interest to the gluon propagator as input for the Dyson-Schwinger equations (DSE) \rightarrow construction of new observables

[C. Fischer, A. Maas, J.A. Müller, Eur. Phys. J. C(2010) 68:165-181]

- Study of the sensitivity of the gluon propagator in pure gauge theory at finite T (as we will see) \implies maybe the construction of a "new" order parameter \implies possibility to move from the confining regime to the perturbative one
- Study of the systematic lattice effects \rightarrow finite volume, small lattice size and gauge fixing effects

Brief Introduction to Lattice Wilson Formulation

- Periodic space-time lattice with lattice spacing a , cf. Ulli Wolf's talk
- Temperature dependence from $T \equiv 1/a \cdot n_\tau$, $n_\tau \ll n_\sigma$
- Gauge field defined as link variables:

$$U_{x,\mu} = e^{ig_0 a A_\mu(x+\hat{\mu}/2)}$$

where: $A(x)$ is the $SU(3)$ gauge field (the gluon field)

- Discretised gauge action for pure gauge theory ($N_c = 3$ for $SU(3)$ and $N_f = 0$)
- In the continuum limit, the Wilson action reduces to $SU(3)$ Yang Mills action S_{YM}

$$S_G(U) = \beta \sum_x \sum_{\mu < \nu} \left(1 - \frac{1}{N_c} \Re \text{tr} U_{x,\mu\nu} \right),$$

$$U_{x,\mu\nu} \equiv U_{x,\mu} U_{x+\hat{\mu},\nu} U_{x+\hat{\mu},\mu}^\dagger U_{x,\nu}^\dagger, \quad \beta \equiv 2N_c/g_0^2$$

$$S_{YM} = \int_0^\beta d\tau \int d^3x \left(-\frac{1}{4} \text{tr} F_{\mu\nu}^a F_a^{\mu\nu} \right), \quad F_{\mu\nu}^a(x) = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_0 f_{abc} A_\mu^b A_\nu^c$$

Symmetries and Implications

- $S_G(U)$ invariant under periodic $SU(3)$ transformations and also global $Z(3)$ center symmetry:

- $A(\vec{x}, \tau) \longrightarrow s(\vec{x}, \tau)(A_\mu(\vec{x}, \tau) + i\partial_\mu)s(\vec{x}, \tau)^\dagger$
- $s(\vec{x}, \beta) = s(\vec{x}, 0)$, with $s(\vec{x}, \beta) = s(\vec{x}, 0)$, and, $s(\vec{x}, \tau) \in SU(3)$
 $s(\vec{x}, \beta) = z s(\vec{x}, 0)$ with $z \in Z(3)$

- Order parameter of the $Z(3)$ symmetry: Polyakov loop

$$\langle L \rangle = \frac{1}{n_\sigma^3} \sum_{\vec{x}} L(\vec{x}), \quad L(\vec{x}) = P \exp(i g_0 \int_0^1 A_4(\vec{x}, \tau) d\tau)$$

- Spontaneous breakdown of $Z(3)$ symmetry at $T > T_c \implies$ signals deconfinement $\implies \langle L \rangle \neq 0$
- A good order parameter for Pure gauge theory (but not anymore in presence of fermions) \implies looking for better order parameters
- Determination of $T_c \rightarrow$ use of the susceptibility:
 $\chi = n_\sigma^3 (\langle L^2 \rangle - \langle |L| \rangle^2)$

Finite T Gluon Propagator in Landau Gauge

- At zero temperature, the gluon propagator in momentum space:

$$D_{\mu\nu}^{ab}(q) = \langle \tilde{A}_\mu^a(k) \tilde{A}_\nu^b(-k) \rangle \equiv \delta^{ab} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) D(q^2)$$

for lattice momenta

$$q_\mu(k_\mu) = \frac{2}{a} \sin \left(\frac{\pi k_\mu}{L_\mu} \right), \quad k_\mu \in (-L_\mu/2, L_\mu/2]$$

- At finite temperature, the gluon propagator $D(p)$ splits up into two structures:

- Transverse propagator:

$$D_T \sim \langle \sum_{i=1}^3 A_i^a(q) A_i^a(-q) - \frac{q_4^2}{q^2} A_4^a(q) A_4^a(-q) \rangle$$

- Longitudinal propagator:

$$D_L \sim \left(1 + \frac{q_4^2}{q^2} \right) \langle A_4^a(q) A_4^a(-q) \rangle$$

- q_4 called the Matsubara frequency
- Understand the behaviour of D_L and D_T with the temperature → expected different behaviour for D_L and D_T

[C. Fischer, A. Maas, J.A. Müller, Eur. Phys. J. C(2010) 68:165-181],[A. Maas, A. Cucchieri, T.

Mendez, hep-lat/0610006, A. Maas, Chin.J.Phys.34:1328-1330,2010]

Setup of the Simulations

- Temperature vs. volume

We study quenched QCD (pure gauge QCD), $48^3 \times n_\tau$, $n_\tau = 4, 6, \dots, 18$ varies

Varying $n_\tau \Leftrightarrow$ varying the temperature

the lattice spacing $a = a(\beta = 6.337) = 0.283 \text{ GeV}^{-1}$ fixed,

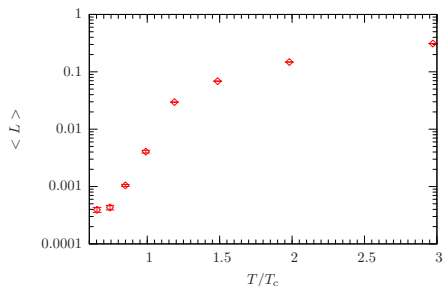
$T_c \longleftrightarrow n_\tau = 12$

- Gauge fixing iteratively to the Landau gauge
- Equivalent to minimize the gauge functional:

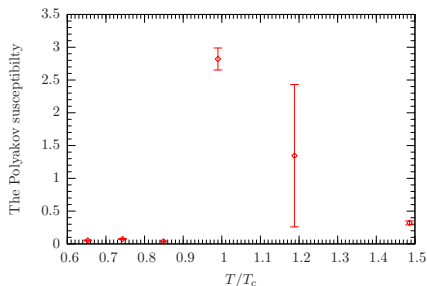
$$F_U(g) = \sum_{x,\mu} \left(1 - \frac{1}{N_c} \Re \text{tr} U_{x\mu}^g \right)$$

$$|(\partial \mathcal{A})_x| = \left| \sum_{\mu=1}^4 (\mathcal{A}_{x+\hat{\mu}/2;\mu} - \mathcal{A}_{x-\hat{\mu}/2;\mu}) \right| < \epsilon \quad \text{for all } x$$

Polyakov Loop and its Susceptibility (Preliminary Results)



(a) The Polyakov loop



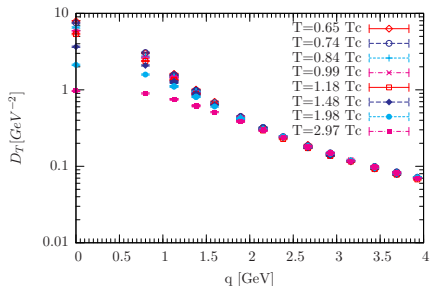
(b) The susceptibility

Figure: Check of phase transition temperature in $SU(3)$ pure gauge theory

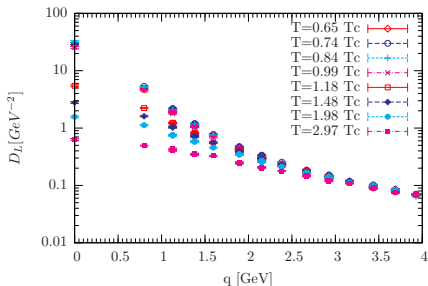
- Rise of the Polyakov loop around the transition temperature
- The susceptibility χ peaks around the phase transition temperature as expected

Temperature Behaviour of Transverse and Longitudinal Propagators

- preselect only (nearly) diagonal momenta \implies to reduce the lattice artifacts
- The renormalisation point: 5 GeV , the Matsubara frequency $q_4 = 0$



(a) The transverse propagator

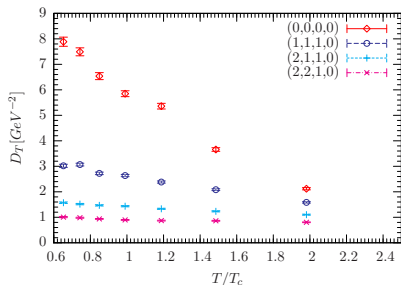


(b) The longitudinal propagator

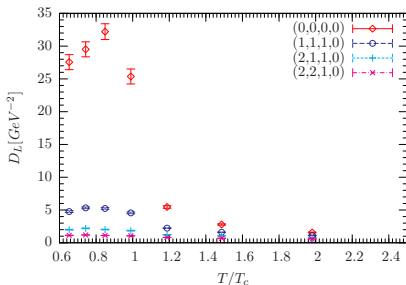
Figure: Temperature behaviour of the transverse and longitudinal propagators

- Big jump in the values of D_L when crossing $T_c \implies$ high sensitivity of D_L to the phase transition
- $D_L \rightarrow$ the same behaviour as an order parameter
- D_T less sensitive

Temperature Behaviour of the Transverse and Longitudinal Propagators at Low momenta



(a) The transverse propagator



(b) The longitudinal propagator

Figure: Temperature behaviour of D_T and D_L at low momenta

- D_L falls down stronger for lower momenta
- Clear sensitivity of $D_L \implies$ possible connection between this behaviour and confinement
- D_T behaves smoothly around T_c

Summary/Outlook

- Results on the gluon propagator at finite T
 - Strong phase transition effect on the longitudinal gluon propagator around T_c
 - Less effect on the transverse gluon observed
- Outlook
 - Including fermions: Unquenched theory ($N_F = 2$)
 - Experimenting new order parameters and possible use of our gluon propagators
 - Study of finite volume and lattice spacing effects \implies scaling tests

Danke für Ihre Aufmerksamkeit